Accelerating transducers

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Overview

- model checking and regular languages
- transducers
- iterating transducers
- conclusion

Model checking

- successful verification technique
- show that M has property φ:

$$\boxed{M \models \varphi}$$

- "push-button"
- via state exploration

$\Rightarrow$ state-space explosion problem

Model checking (cont'd)

- specifically nasty[1] instance of too big state space: infinite many states
- reasons: infinite data, infinite control (e.g. parameterized systems), time . . .
- scores of approaches:
  - use your own brain (and your own time . . .): theorem proving
  - abstraction
  - symbolic techniques (many):

    > symbolic = don't explore states one-by-one, but represent sets of states "symbolically" and explore them all at the same time

- 3 questions:
  1. how to represent infinite sets of states
  2. how to represent the reduction relation?
  3. how to calculate the reachable states in a finite amount of time?

[1] and quite common, for that matter

Regular model checking

- *very* successful finite description/symbolic representation of infinite "objects": regular languages

⇒ regular model checking (e.g., for parameterized systems $P_1 \parallel P_2 \parallel \ldots$, (cf. [7][9][1][8]. . .):

represent

- local state as letters of an alphabet
- global states as linear arrangement of local ones = word

⇒ infinite sets of states = reg. language

⇒ computation step, i.e., non-det. change of language = transduction

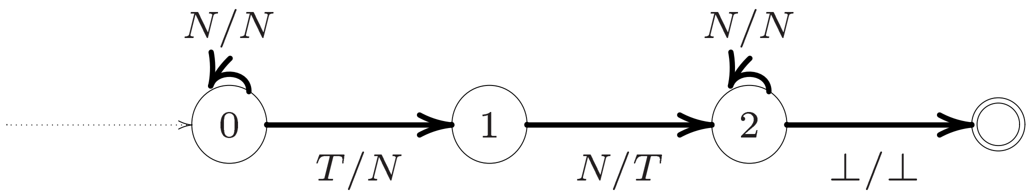
Example

Example 1. [Token array] *"Parameterized" processes: each one either has the token or not (states T and N). Token can be passed between neighbors from left to right, initially, the token is owned by the left-most process.*

Initial configuration:	TN^*
one step:	$TN \to NT$

Example (cont'd)

- Effect of one-step reduction relation: captured by a transducer



- *e.g.:* $\mathcal{T}(NTNN) = \{NNTN\}$

$\Rightarrow$ exploit for symbolic exploration: $\mathcal{T}^n \circ \mathcal{A}$

$$\begin{aligned} &= \{t' \in \mathcal{T}^n(t) \mid \text{and } t \text{ accepted by } \mathcal{A}\} \\ &= \{t' \mid t \rightarrow^n t', t \text{ accepted by } \mathcal{A}\} \end{aligned}$$

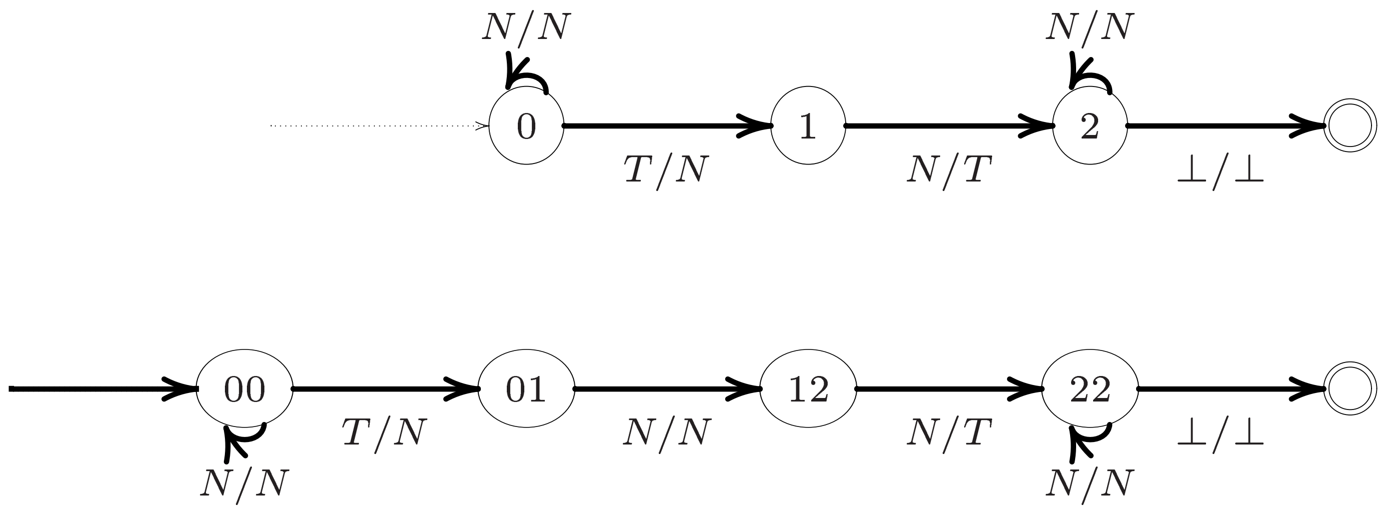
Goal: iterating transducers

- assuming that you know how to calculate $T_1 \circ \mathcal{T}_2$ by a product construction: calculate $\mathcal{T}^*$ as fixpoint $\mu X.\mathcal{T} \circ (X \cup \mathcal{T}_{id})$?, but

  1. $\mathcal{T}^*$ may not be representable as finite transducer (e.g.: duplicating the number of letter a: $q_0 a(x) \rightarrow a a q_0(x)$)

  2. even if: calculating $\mu X.\mathcal{T} \circ (X \cup \mathcal{T}_{id})$ iteratively will in general diverge following page

Example: first 2 iterations



A finite representation for $\mathcal{T}^*$?

- a sound infinite representation $\mathcal{T}^{<\omega}$ for $\mathcal{T}^*$ is straighforward (using Q^* as set of states)

$\Rightarrow$ for a finite representation: build a quotient $\mathcal{T}^{<\omega}_{/\cong}$

$\Rightarrow$ remains:

1. What to take for $\cong$?
2. How to compute $\mathcal{T}^{<\omega}_{/\cong}$?

Key observation for quotienting

Theorem 2. [Soundness] *given* $F, P \subseteq Q^*$

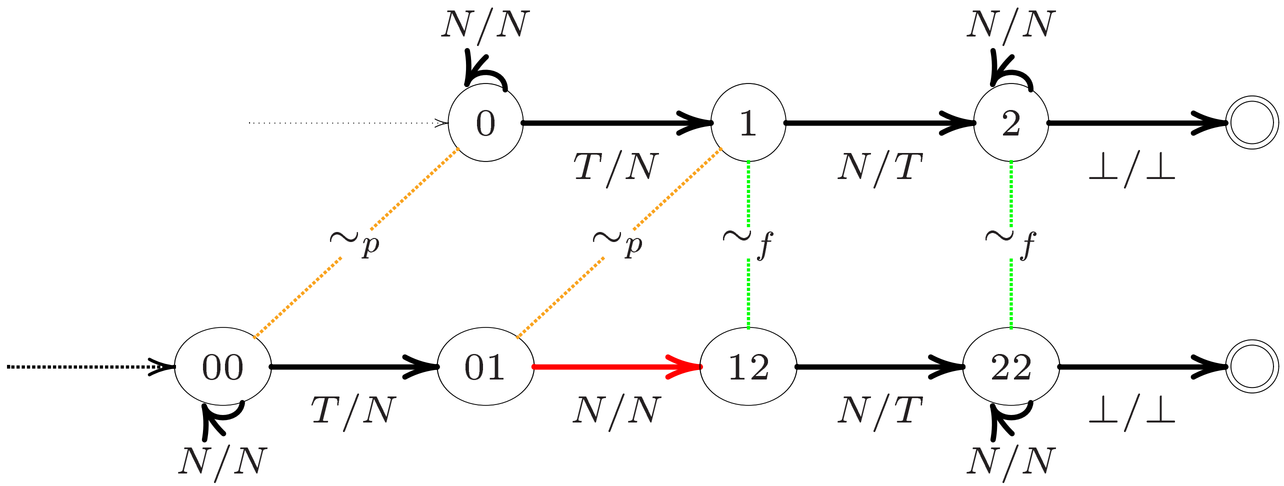
- F *and* P *two bisimulations (future and past)*
- F *and* P *swap, meaning that*

$$F; P = P; F$$

$\Rightarrow$

$$\boxed{[\![\mathcal{T}^{<\omega}]\!] = [\![\mathcal{T}^{<\omega}_{/F;P}]\!]}$$

Example, revisted



$q_{01} \sim_p q_1 \sim_f q_{12}$

But still: how to *compute* $\mathcal{T}^{<\omega}_{/F;P}$?

$\mathcal{T}^{<\omega}$ is infinite! (for Q^* is)

- way out:
  - calulate bisim's P and P on finite appoximations $\mathcal{T}^{\leq n}$
  - "extrapolate" to $\mathcal{T}^{<\omega}$

- How to extrapolate?

$\Rightarrow$ use rewriting theory, replace P and F by $\leftrightarrow^*_F$ and $\leftrightarrow^*_P$.

  - bisimulations are congruences wrt. to the monoid Q^*
  - extrapolate swapping condition (for instance): if $\leftrightarrow_P$ and $\leftrightarrow_F$ are confluent and swap, then so are $\leftrightarrow^*_P$ and $\leftrightarrow^*_F$.

$\Rightarrow$ bisimulations found in finite $\mathcal{T}^{\leq n}$ can be used to quotient $\mathcal{T}^{<\omega}$

Algorithm

```
input  𝒯 = (Q, Q_0, Σ, R)
𝒳 = 𝒯_id ;
repeat
   𝒳 := (𝒯 ∘ 𝒳) ∪ 𝒯_id ;
   determine  bisimulations   F and P on 𝒳 s.t.
      ↔_F and ↔_P swap and each possess the diamond
      property ;
until  𝒳_/≡ ∼_f (𝒯 ∘ 𝒳_/≡) ∪ 𝒯_id
```

Math form of the listing:

input $\mathcal{T} = (Q, Q_0, \Sigma, R)$

$\mathcal{X} = \mathcal{T}_{id}$;

repeat

$\mathcal{X} := (\mathcal{T} \circ \mathcal{X}) \cup \mathcal{T}_{id}$;

determine bisimulations F and P on $\mathcal{X}$ s.t. $\leftrightarrow_F$ and $\leftrightarrow_P$ swap and each possess the diamond property ;

until $\mathcal{X}_{/\equiv} \sim_f (\mathcal{T} \circ \mathcal{X}_{/\equiv}) \cup \mathcal{T}_{id}$

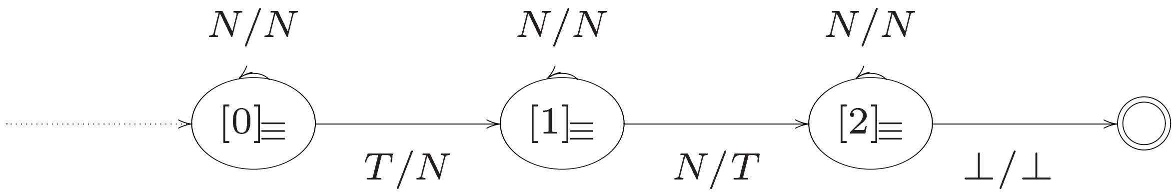
Example

- Rewrite system after 2 iterations:

$$\begin{aligned} 00 &\rightarrow 0 \\ 01 &\rightarrow 1 \\ 12 &\rightarrow 1 \\ 22 &\rightarrow 2 \end{aligned}$$

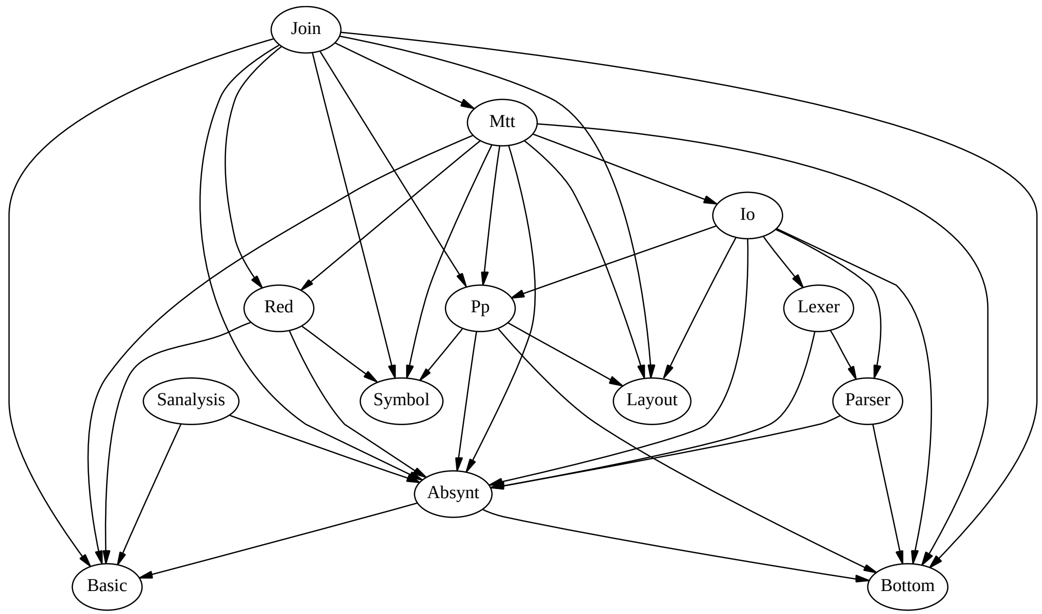
i.e.

$[0] = \{0, 00, \ldots\}$,
$[1] = \{1, 01, 001, \ldots, 12, 122, \ldots\}$,
$[2] = \{2, 22, \ldots\}$.



Implementation

- library of transducer-operations (iteration, composition, transduction)
- in *ocaml*
- efficiency: sufficient for small examples



Conclusion and further directions

- characterize iterable transducers, complexity?
- ϵ-transitions and weak bisimulation?
- Compare with
  - monadic string rewriting [3]
  - column-transducers of k-bounded depth [9]
- possible to specialize: $\mathcal{T}^{\leq n} \circ \mathcal{A}$. The construction carries over? Does one benefit from that?
- more complicated examples, dynamic process creation
- implementation: efficiency, various optimizations
- further into the jungle of tree transducers[2] . . .

[2]for tackling data, one needs trees not just words.

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