

# Operational Semantics of a Weak Memory Model with Channel Communication

Daniel Fava, Martin Steffen, Volker Stolz

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# In this talk

- *operational* semantics for a WMM
  - inspired by Go
  - channel communication
  - based on happens-before
- proof of basic *correctness* property
- executable within the  $\mathbb{K}$  rewriting framework



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# Memory model



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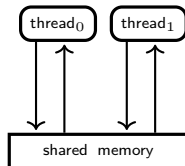
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## MCM

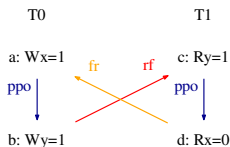
A specification what to expect from from a shared memory, what may be observed (by reads) and what not.

- bottom-line: *sequential consistency* Lamport (interleaving of reads and writes),
- *weak* or *relaxed*: basically weaker than that.



# How to specify a MM

- in prose (as in <https://golang.org/ref/mem>)
- litmus tests
- **axiomatic** (candidate executions)
- **operational** (SOS)



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# Go sales pitch



- “language for the 21st century”
- relatively new language (with some not so new features?)
- a lot of *fanfare* & backed by Google no less
- existing show-case applications
  - docker
  - dropbox . . .



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# Go's stated design principles

- appealing to C programmers
- KISS: “keep it simple, stupid”
- built-in **concurrency**
- “strongly typed”
- efficient
- fast *compilation*, appealing for scripting



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# Go's non-revolutionary feature mix

- imperative
- object-oriented (?)
- compiled
- concurrent (goroutines)
- “strongishly” typed
- garbage collected
- portable
- higher-order functions and closures



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- simple concurrent calculus with “goroutines”
- A-normal form
- channels:
  - dynamically created
  - “higher-order” channels (à la  $\pi$  ...)
  - *bounded* channels
  - mixed choice
    - with “channel guards”
    - default-clause



# Syntax

$v ::= r \mid \underline{n}$	values
$e ::= t \mid v \mid \text{load } z \mid z := v \mid \text{if } v \text{ then } t \text{ else } t \mid \text{go } t$ $\mid \text{make}(\text{chan } T, v) \mid \leftarrow v \mid v \leftarrow v \mid \text{close } v$	expression
$g ::= v \leftarrow v \mid \leftarrow v \mid \text{default}$	guards
$t ::= \text{let } r = e \text{ in } t \mid \sum_i \text{let } r_i = g_i \text{ in } t_i$	threads

- $\sum$  : choice (select, case, default)

# Go's concurrency model

- **only** sync-primitive: *channel* communication (but read the fine-print)
- shared variable communication possible  $\Rightarrow$
- simple **happens-before** memory model

## Mantra

Don't communicate by sharing memory; share memory by communicating. (R. Pike)

- straightforward and simple model, still they advise:  
*"If you must read the rest of this document [= the Go MM] to understand the behavior of your program, you are being too clever. Don't be clever."*



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# Happens-before

- dates back to Lamport
- “unrelated” to actually “happening before”

## Observational + “liberal”

a read **can observe** a write  $W$  **unless**

1. read **definitely** “too late”
2. a different write **definitely** “overwrites”  $W$  (shadows)



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# Nature of synchronization

- generally:<sup>1</sup> restricting otherwise possible *interleavings*
- in connection with shared memory: intuition often (cf. *write buffers*)

*“Data Memory Barrier (DMB). This forces all earlier-in-program-order memory accesses to **become** globally **visible** before any subsequent accesses.”*  
(random quote, some ARM programmer’s guide)



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<sup>1</sup>independent from shared memory

# Nature of synchronization

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*“Data Memory Barrier (DMB). This forces all earlier-in-program-order memory accesses to **become** globally **visible** before any subsequent accesses.”*  
*(random quote, some ARM programmer’s guide)*

## Happens-before

*synchronization* = making things **INVISIBLE**



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<sup>1</sup>independent from shared memory

# Two(\*) ingredients for HB only

1. *program* order
2. channel communication
  - 2.1 sending  $\rightarrow_{hb}$  receiving
  - 2.2 full buffer

## Sends and receives

- A *send* on a channel happens-before the corresponding *receive* from that channel completes.
  - The  $i$ th *receive* on a channel with capacity  $k$  happens-before the  $i + k$ th *send* from that channel completes.
- 
- channel close, init, thread creation, packages, locks, once



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# Operational semantics (weak)



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## Configuration

$$P ::= p\langle\sigma, t\rangle \mid m(z:=v) \mid c[q] \mid \bullet \mid P \parallel P \mid \nu n P \quad (1)$$

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# Operational semantics (weak)



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## Configuration

$$P ::= p\langle\sigma, t\rangle \mid m(z:=v) \mid c[q] \mid \bullet \mid P \parallel P \mid \nu n P \quad (1)$$

## thread

$$n\langle\sigma, t\rangle$$



# Operational semantics (weak)



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## thread

$$n\langle\sigma, t\rangle$$

## channel

$$n[q]$$

# Operational semantics (weak)



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## Configuration

$$P ::= p\langle\sigma, t\rangle \mid m(z:=v) \mid c[q] \mid \bullet \mid P \parallel P \mid \nu n P \quad (1)$$

thread	write event	channel
$n\langle\sigma, t\rangle$	$n(z:=v)$	$n[q]$

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## Write & read steps

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$$\frac{\dots}{p\langle\sigma, z := v; t\rangle \rightarrow p\langle\sigma', t\rangle \parallel n(z:=v)}$$

$$\frac{\dots}{p\langle\sigma, \text{let } r = \text{load } z \text{ in } t\rangle \parallel n(z:=v) \rightarrow p\langle\sigma, \text{let } r = v \text{ in } t\rangle \parallel n(z:=v)}$$

---

# Synchronization = making things unobservable

- reads and writes: no **synchronization**
- **program order**
  - the only component of happens-before
  - for  $x := 1; \quad x := 2$ : value 1 unobservable

but only locally

- channel communication; only (interesting) means of synchronization



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## Channel communication

- send the communicated value from sender to receiver



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  - for  $x:=1; \quad x:=2$ : value 1 unobservable

but only locally

- channel communication; only (interesting) means of synchronization

## Channel communication

- send the communicated value from sender to receiver
- inform receiver of local knowledge of **UNobservable** write events



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# Shadow sets and local information

- every write even: unique identifier

## thread local information

1. which events are locally **known** to be unobservable (**shadowed**)
2. which events are locally **known** to have **happened-before** (at the current point)

$$n\langle\sigma, t\rangle$$

- local “state” tuple  $(E_{hb}, E_s), \sigma : 2^{(N \times X)} \times 2^N$ .



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## Read & write once more

- of course: shadow sets used to make writes invisible
- local update of “program order”

---

$$\sigma = (E_{hb}, E_s) \quad \sigma' = (E_{hb} + (\mathbf{n}, \mathbf{z}), E_s + \mathbf{E}_{hb}(\mathbf{z}))$$

$$\frac{}{p\langle \sigma, z := v; t \rangle \rightarrow p\langle \sigma', t \rangle \parallel n(z := v)}$$

$$\sigma = (\_, E_s) \quad \mathbf{n} \notin \mathbf{E}_s$$

---

$$p\langle \sigma, \text{let } r = \text{load } z \text{ in } t \rangle \parallel n(z := v) \rightarrow p\langle \sigma, \text{let } r = v \text{ in } t \rangle \parallel n(z := v)$$

---



# Channel communication

- sending values + knowledge about  $\sigma$

---

$$\frac{\neg \text{closed}(c_f[q_2]) \quad \sigma' = \sigma}{p\langle\sigma, c \leftarrow v; t\rangle \parallel c[q_2] \rightarrow p\langle\sigma', t\rangle \parallel c[(v, \sigma) :: q_2]} \text{R-SEND}$$

$$\frac{v \neq \perp \quad \sigma' = \sigma + \sigma''}{\begin{array}{l} c_b[q_1] \parallel p\langle\sigma, \text{let } r = \leftarrow c \text{ in } t\rangle \parallel c_f[q_2 :: (v, \sigma'')] \rightarrow \\ c_b[\sigma :: q_1] \parallel p\langle\sigma', \text{let } r = v \text{ in } t\rangle \parallel c_f[q_2] \end{array}} \text{R-REC}$$

---

# Bounded channels

- A *send* on a channel happens-before the corresponding *receive* from that channel completes.
- The  $i$ th *receive* on a channel with capacity  $k$  happens-before the  $i + k$ th *send* from that channel completes.



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- The  $i$ th *receive* on a channel with capacity  $k$  happens-before the  $i + k$ th *send* from that channel completes.

## Bounded channels

- . There is also a “backward synchronization”
  - from an “earlier” receive to a sender
- *forward channel* (as shown)
- *backward channel*, propagating local  $\sigma$  knowledge



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## Send and receive

---

$$\frac{\neg \text{closed}(c_f[q_2]) \quad \sigma' = \sigma + \sigma''}{c_b[q_1 :: \sigma''] \parallel p\langle \sigma, c \leftarrow v; t \rangle \parallel c_f[q_2] \rightarrow c_b[q_1] \parallel p\langle \sigma', t \rangle \parallel c_f[(v, \sigma) :: q_2]} \text{R-SEND}$$

$$\frac{v \neq \perp \quad \sigma' = \sigma + \sigma''}{c_b[q_1] \parallel p\langle \sigma, \text{let } r = \leftarrow c \text{ in } t \rangle \parallel c_f[q_2 :: (v, \sigma'')] \rightarrow c_b[\sigma :: q_1] \parallel p\langle \sigma', \text{let } r = v \text{ in } t \rangle \parallel c_f[q_2]} \text{R-REC}$$

---



$$\sigma' = \sigma_1 + \sigma_2$$

$$\begin{array}{l} c_b[] \parallel p_1\langle\sigma_1, c \leftarrow v; t\rangle \parallel p_2\langle\sigma_2, \text{let } r = \leftarrow c \text{ in } t_2\rangle \parallel c_f[] \rightarrow \\ c_b[] \parallel p_1\langle\sigma', t\rangle \parallel p_2\langle\sigma', \text{let } r = v \text{ in } t_2\rangle \parallel c_f[] \end{array}$$

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# Delayed reads

- so far: delayed or buffered writes
- also *delayed reads* (load buffers) possible  $\Rightarrow$  “read events”

## More (and more complex) “events”

$$m(\sigma, z := n_1)_p \quad \text{and} \quad m(\sigma, ?n_4)_p$$

- chain of “future references”
- symbolic execution
- nota bene:
  - the write itself is not “delayed”
  - it’s the negative information (invisibility of other writes via the shadow sets, that travels slow)



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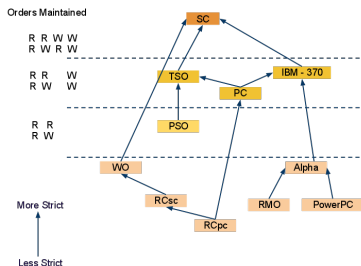
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- WMMs, it's a jungle ...
- out-of-thin air
  - should be avoided (or should it?)
  - not even crystal clear what it is.



... but there's a bottom line

No matter how “relaxed” you want your memory model one thing is non-negotiable:

## DRF-SF

Data-race free programs have to be sequentially consistent  
Manson et al. [8]



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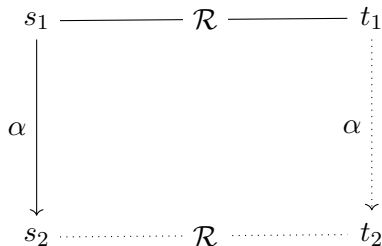
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# Simulation



**“weak simulates strong”**

sure thing



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# Simulation



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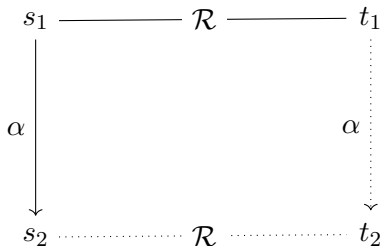
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**“weak simulates strong”**

sure thing

**“strong simulates weak”**

definitely not

# Simulation



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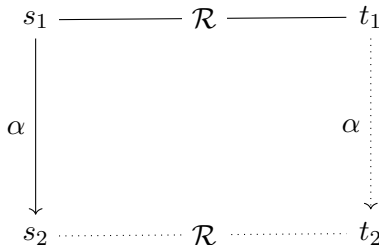
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**“weak simulates strong”**

sure thing

**“strong simulates weak”**

conditionally, for RF programs

- “simultaneous” access to a shared location, where at least one is a write access

## Manifest race (case W/W)

config  $C$  with

$$C \xrightarrow{(z!)p_1} s \xrightarrow{(z!)p_2} s$$

- race: reachable configuration with manifest race

# Core of the proof



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## abstraction function/relation

“weak config”  $\rightarrow$  “strong config”

- problem: configs contains “alternatives”

$$n_1(|z:=v_1|) \parallel n_2(|z:=v_2|)$$

- strong semantics: exactly **one** value of  $z$

# From local to global view $\Rightarrow$ consensus



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## Lemma (Consensus possible)

*Weak configurations obey the following invariant*

$$\bigcap_{p \in P} W_P^o(z @ p) \neq \emptyset . \quad (2)$$

- adding also *read events* to configurations

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# RF programs $\Rightarrow$ stronger consensus

## Lemma (Race-free consensus when it counts)

Assume  $P_0 \rightarrow_w^* P$  with  $P_0$  race-free. If  $P \xrightarrow{(z?)_p}_w$  or  $P \xrightarrow{(z!)_p}_w$ , then there exists a write event  $m(|z:=v|)$  such that

$$\bigcap_{p_i} W_P^o(z@p_i) = \{m\} , \quad (3)$$

where the intersection ranges over an arbitrary set of processes which includes  $p$ .

## Lemma (Race-free consensus)

Weak configurations for race-free programs obey the following invariant

$$\bigcap_{p_i \in P} W_P^o(z@p_i) = \{m\} \quad (4)$$

for some write event  $m(|z:=v|)$ .



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# Conditional simulation



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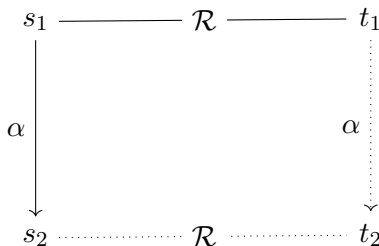
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- augment the configuration with additional read-events
- ⇒ consensus lemmas
- ⇒ DRF-SC



- rewrite-based engine
- used variously for executable semantics
  - C++ memory models
  - “Ethereum” smart contracts platform
  - ...
- see <https://github.com/dfava/mmgo>



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# Receive in $\mathbb{K}$

## R-Rec

$$v \neq \perp \quad \sigma' = \sigma + \sigma''$$

$$\frac{c_b[q_1] \parallel p\langle\sigma, \text{let } r = \leftarrow c \text{ in } t\rangle \parallel c_f[q_2 :: (v, \sigma'')] \rightarrow c_b[\sigma :: q_1] \parallel p\langle\sigma', \text{let } r = v \text{ in } t\rangle \parallel c_f[q_2]}{}$$

---

```
rule <goroutine>
  <k> <- channel(Ref:Int) => V ... </k>
  <sigma>
    <HB> HMap:Map => mergeHB(HMap, HMapDP) </HB>
    <S> SSet:Set => SSet SSetDP </S>
  </sigma>
  <id> _ </id>
</goroutine>
<chan>
  <ref> Ref </ref>
  <type> _ </type>
  <forward> ListItem( ListItem(V)
    ListItem(HMapDP)
    ListItem(SSetDP) ) => .List </forward>
  <backward> BQ:List => ListItem( ListItem(HMap)
    ListItem(SSet)) BQ </backward>
</chan>
requires notBool( V ==K $eot )
```

---

# Related work

- loads of material on *axiomatic* semantics
- operational:
  - Boudol and Petri based on *rewriting theory*
  - Kang et al.: “promising” semantics with “*clocks*”
  - Flanagan and Freund: *adversarial* memory
  - Demange et al. Plan B (Java buffered write semantics BMM)
  - Pichon-Pharabod and Sewell: operational semantics avoiding OOTA
  - Alrahman et al.
  - Matthias Perner et al: parametrized semantics for NI (earlier today)
  - ...



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# Conclusion

- formalizing WMM for some calculus with channel
- DRF-SC simulation proof
- read delays under work



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**Operational  
Semantics of a  
Weak Memory  
Model with  
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**Introduction**

**Calculus**

**Correctness**

**Conclusion**