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## Abstract

Actor-based languages attract attention for their ability to scale to highly parallel architectures. Active objects combine the asynchronous communication of actors with object-oriented programming by means of asynchronous method calls and synchronization on futures. However, the combination of asynchronous calls and synchronization may introduce communication cycles which lead to a form of communication deadlock and livelock. This paper addresses such communication deadlocks and livelocks for ABS, a formally defined active object language which additionally supports cooperative scheduling to express complex distributed control flow, using first-class futures and explicit process release points. Our approach is based on a translation of the semantics of ABS into colored Petri nets, such that a program and its state correspond to a marking of this net. We prove the soundness of this translation and demonstrate by example how the implementation of this net can be used to analyze ABS programs with respect to communication deadlocks and livelocks.

*Keywords:* Colored Petri nets, modelling, model checking, semantics, static analysis, concurrency

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## 1. Introduction

The Actor model [1, 2] of concurrency is attracting increasing attention for its *decoupling* of control flow and communication. This decoupling enables both scalability (as argued with the Erlang programming language [3] and Scala’s actor model [4]) and compositional reasoning [5–7]. Actors are independent units of computation which exchange messages and execute local code sequentially. Instead of pushing the current procedure (or method activation) on the control stack when sending a message as in thread-based concurrency models, messages are sent asynchronously, without any transfer of control between the actors. In the actor model, a message triggers the execution of a method body in the target actor, but a reply to the message is not directly supported. Extending the basic actor model, active object languages (e.g., [8–10]) combine actor-like communication with object orientation, use so-called futures to reintroduce synchronization by combining asynchronous message sending with the call and reply structure of method calls. A future can be seen as a mailbox from which a reply may be retrieved, such that the synchronization is decoupled from message

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sending and associated with fetching the reply from a method call. The caller synchronizes with the existence of a reply from a method call by performing a *blocking* get-operation on the future associated with the call. However, this synchronization may lead to complex dependency cycles in the communication chain of a program, and gives rise to a form of deadlock with a set of mutually blocked objects. This situation is often called a *communication deadlock* [11].

In this paper we work with the active object language ABS [9, 12]. ABS is characteristic in that it supports *cooperative concurrency* in the active objects. Cooperative concurrency allows the execution of a method body to be suspended at explicit points in the code, for example by testing whether a future has received a value. Cooperative concurrency leads to a form of local race-free interleaving for concurrently executing active objects, which allows more execution traces than in standard active objects. However, while no progress is made at the suspension points, the scheduler is continuously activating and releasing processes. We call this situation *communication livelock*, in the sense that the object is not blocked, but the generated processes can not progress due to that each of them is busy-waiting for a process on another object.

This paper addresses the problem of communication deadlock and livelock for the active object language ABS. Our approach to tackle these problems is based on a translation of the formal semantics of ABS into colored Petri nets (CPN) [13]. Petri nets provide a basic model of concurrency, causality, and synchronization [14, 15], which has previously been used to analyze communication patterns and deadlock (e.g., [16, 17]). CPNs extend the basic Petri net model with support for modeling data. In contrast to previous work, we do not produce a different Petri net for each ABS program to be analyzed. Instead, we provide an encoding and implementation of the formal semantics of ABS itself as a net, and use colored tokens in this net to encode the program and its state. Consequently, the number of places in the net is independent of the size of a program, and different programs are captured by different markings of the net. This also allows us to capture dynamic object creation by firing transitions in the net. The main contributions of this paper are:

- an encoding of the formal semantics of ABS in CPNs;
- a translation of concrete ABS programs into markings of this net;
- a soundness proof for the translation from ABS to CPN; and
- a case study demonstrating how to analyze communication deadlocks and livelocks for active objects in ABS using the implementation of this net in CPN Tools [18].

This paper extends a paper from FSEN 2017 [19]; compared to that paper we here present the translation of ABS into CPN and the associated correctness proof in full detail, and add support for livelock analysis. The communication analysis has also been improved to be directly supported in the CPN Tools.

The paper is organized as follows: Section 2 introduces the ABS language, focusing on language features for communication and synchronization, and Section 3 introduces colored Petri nets. Section 4 discusses the translation from ABS semantics to colored Petri nets and Section 5 the translation from ABS configurations to Petri net markings, before Section 6 considers the soundness of the translation. Section 7 presents ABS examples and show how the CPN Tool detects communication deadlock and livelock, respectively. Finally, Section 8 concludes the paper with a discussion of related and future work.

## 2. The ABS Concurrency Model

The Abstract Behavioral Specification language (ABS) [9, 12] is an object-oriented language for modeling concurrent and distributed systems. ABS is an active object language [10] which combines asynchronous communication from the Actor model [1, 2] with object orientation, and supports cooperative scheduling such that process release points are explicit in the program code.

For the purposes of this paper, we focus on the communication and synchronization aspects of ABS. Also we ignore other aspects such as concurrent object groups (i.e., we consider one object per group), the functional sublanguage, and deployment aspects such as deployment components and resource annotations [12]. ABS is statically typed, based on interfaces as object types [9]. Ignoring the details of the type system, we let primitive types such as `Int` and `Bool` and class names constitute the types of a program, and ignore subtyping issues.

The above simplifications make the translation of ABS into colored Petri nets (which we will discuss in detail in the rest of the paper) lead to coarser program over-approximations. It is possible to add these details to the translation in a similar way, but in the current work we are interested in the communication topologies of ABS rather than in the full functional details as our focus is on communication analysis and the detection of deadlocks and livelocks.

### 2.1. The Syntax

Figure 1 presents the syntax of ABS [9], focusing on communication and synchronization. Programs  $P$  consist of class definitions `CL` and a main block representing the program's initial activity. Statements  $s$  include standard control-flow constructs such as sequential composition, assignment statement, conditionals, and while-loops. ABS supports *asynchronous* method calls  $f = e!m(\bar{e})$  where the caller and callee proceed concurrently and  $f$  is a so-called *future*. A future is a “mailbox” where the return value from the method call may eventually be returned to by the callee. A future that contains a return value is resolved. The result of the asynchronous call can then be obtained by  $f$ . **get**. Note that we may alternatively write asynchronous method call statement as  $e!m(\bar{e})$ , if the return value is not required. ABS also supports local synchronous calls which synchronize in the usual reentrant way, passing control directly to the called method and then back to the callee upon completion.

The (active) objects of ABS act like monitors, allowing at most one method activation, or process, to be executed at a time. The local execution in an object is based on *cooperative* scheduling by introducing a guard statement **await**  $g$ : If  $g$  evaluates to true, execution may proceed; if the guard  $g$  evaluates to false, execution is suspended and another process may execute. For a future  $f$ , the guard  $f?$  evaluates to true if  $f$  contains the return value from the associated method call and otherwise it evaluates to false. The **suspend** statement always suspends the executing process. The typical usage of asynchronous calls follow the pattern  $f = e!m(\bar{e}); \dots; \mathbf{await} \ f?; \dots; x = f.\mathbf{get}$ .

### 2.2. The Operational Semantics

The operational semantics of ABS specifies transitions between *configurations*. A run-time configuration contains objects  $o(a, p, q)$ , messages  $\langle o'.m(v) \rangle_f$ , resolved futures  $\langle v \rangle_f$ , and unresolved futures  $\langle \perp \rangle_f$ . We use  $\parallel$  to denote the (associative and commutative) parallel composition of such entities in a run-time configuration. Class definitions (which do not change during execution), are assumed to be implicitly available in the operational rules. The semantics maintains as invariant that object identities  $o$  and future identities  $f$  are unique. Objects  $o(a, p, q)$  are instances of classes with an identifier  $o$ , an object state  $a$  which maps instance variables to values, an active process

<i>Syntactic categories.</i>	<i>Definitions.</i>
$s$ in Stmt	$P ::= \overline{CL} \{ \overline{T} \ \overline{x}; \ s \}$
$e$ in Expr	$CL ::= \mathbf{class} \ C \ (\overline{T} \ \overline{x}) \{ \overline{T} \ \overline{x}; \ \overline{M} \}$
$g$ in Guard	$Sg ::= T \ m \ (\overline{T} \ \overline{x})$
	$M ::= Sg \ \{ \overline{T} \ \overline{x}; \ s \}$
	$s ::= s; s \mid \mathbf{skip} \mid x = rhs \mid \mathbf{if} \ e \ \{s\} \ \mathbf{else} \ \{s\}$
	$\mid \mathbf{while} \ e \ \mathbf{do} \ s \mid \mathbf{suspend} \mid \mathbf{await} \ g \mid \mathbf{return} \ e$
	$rhs ::= e \mid cm \mid \mathbf{new} \ C(\overline{e})$
	$cm ::= e!m(\overline{e}) \mid x. \mathbf{get}$
	$g ::= x? \mid g \wedge g$

Figure 1: Abstract syntax of ABS, where overline notation such as  $\overline{e}$  and  $\overline{x}$  denotes (possibly empty) lists over the corresponding syntactic categories.

$p$ , and an unordered queue  $q$  of suspended processes that are candidates to be activated by the scheduler if the currently active process will suspend or return. A *process*  $p$  is a triple  $\langle l \mid s \rangle_f$  with a local state  $l$  (mapping method-local variables to values), a statement  $s$ , and a future reference  $f$ . We omit the future reference in the rules if it is unnecessary. The special process *idle* is used to represent that there is no active process. A message  $\langle o'.m(v) \rangle_f$  represents a method call *before* it starts to execute and the resolved future  $\langle v \rangle_f$  the corresponding return value after method execution.

Figure 2 gives the rules of the operational semantics, concentrating on the behavior of a single active object. A **skip** statement has no effect (cf. rule SKIP). In an idle object, the scheduler selects (and removes) a process  $p$  from the queue, and starts executing it (cf. rule ACTIVATE). Executing **suspend** moves the active process to the queue, resulting in an idle object (cf. rule SUSPEND). ASSIGN<sub>1</sub> and ASSIGN<sub>2</sub> are the assignment rules. Assignments are either to instance variables or local variables, where  $\sigma$  is used to abbreviate the pair of local states  $l$  and object states  $a$ . We assume that these are disjoint, so the two cases are mutually exclusive. We omit the standard rules for conditionals and while-loops.

Object creation is captured by the NEW-OBJECT rule, where  $a'$  is the initial state of the new object (determined by an auxiliary function *atts*) and  $p'$  is the object's initial activity. An asynchronous method call creates a fresh future reference  $f$  and adds a message and unresolved future corresponding to the call to the configuration (cf. rule ASYNC-CALL). Binding a method name to the corresponding method body is done in rule BIND-MTD. The binding operation, locating the code of the method body and instantiating the formal parameters, works in the standard way via late-binding, consulting the class hierarchy. The return statement stores the return value in the corresponding future, resolving the future (cf. rule RETURN).

The **get** statement allows the result value to be obtained from the corresponding future reference if the future's value has been produced, in which case the future has been *resolved* (cf. rule GET). Otherwise, the **get** statement blocks. An attempt to fetch a future value via a **get** statement does not introduce a scheduling point. Should the value never be produced, e.g., because the corresponding method activation does not return, the client object of the future, executing the **get** statement, will be blocked. A common pattern for obtaining a future value therefore makes use of **await**: executing **await**  $x?; x. \mathbf{get}$  checks whether or not the future reference for variable  $x$  has been produced. If not, the semantics of the **await** statement introduces a scheduling point. Once the guard  $x?$  evaluates to true, the future's value remains available so  $x. \mathbf{get}$  will not block (see again rule READ-FUT). Executing an **await** with a guard expression which evaluates to the

$$\begin{array}{c}
\text{(SKIP)} \\
\frac{}{o\langle a, \langle l \mid \mathbf{skip}; s \rangle, q \rangle \rightarrow o\langle a, \langle l \mid s \rangle, q \rangle} \\
\\
\text{(ACTIVATE)} \\
\frac{p = \mathit{select}(q, a)}{o\langle a, \mathit{idle}, q \rangle \rightarrow o\langle a, p, q \setminus p \rangle} \\
\\
\text{(SUSPEND)} \\
\frac{}{o\langle a, \langle l \mid \mathbf{suspend}; s \rangle, q \rangle \rightarrow o\langle a, \mathit{idle}, \langle l \mid s \rangle :: q \rangle} \\
\\
\text{(ASSIGN}_1\text{)} \\
\frac{x \in \mathit{dom}(l)}{o\langle a, \langle l \mid x = e; s \rangle, q \rangle \rightarrow o\langle a, \langle l[x \mapsto \llbracket e \rrbracket_\sigma] \mid s \rangle, q \rangle} \\
\\
\text{(ASSIGN}_2\text{)} \\
\frac{x \in \mathit{dom}(a)}{o\langle a, \langle l \mid x = e; s \rangle, q \rangle \rightarrow o\langle a[x \mapsto \llbracket e \rrbracket_\sigma], \langle l \mid s \rangle, q \rangle} \\
\\
\text{(BIND-MTD)} \\
\frac{p = \mathit{bind}(o, m, \bar{v}, f)}{o\langle a, \langle l \mid s \rangle, q \rangle \parallel \langle o.m(\bar{v}) \rangle_f \rightarrow o\langle a, \langle l \mid s \rangle, p :: q \rangle} \\
\\
\text{(NEW-OBJECT)} \\
\frac{\mathit{fresh}(o') \quad a' = \mathit{atts}(C, \llbracket \bar{e} \rrbracket_\sigma, o')}{o\langle a, \langle l \mid x = \mathbf{new} \ C(\bar{e}); s \rangle, q \rangle \rightarrow o\langle a, \langle l \mid x = o'; s \rangle, q \rangle \parallel o'\langle a', \mathit{idle}, \emptyset \rangle} \\
\\
\text{(ASYNC-CALL)} \\
\frac{\llbracket e \rrbracket_\sigma = o' \quad \mathit{fresh}(f)}{o\langle a, \langle l \mid x = e!m(\bar{e}); s \rangle, q \rangle \rightarrow o\langle a, \langle l \mid x = f; s \rangle, q \rangle \parallel \langle o'.m(\llbracket \bar{e} \rrbracket_\sigma) \rangle_f \parallel \langle \perp \rangle_f} \\
\\
\text{(RETURN)} \\
\frac{}{o\langle a, \langle l \mid \mathbf{return} \ (e); s \rangle_f, q \rangle \parallel \langle \perp \rangle_f \rightarrow o\langle a, \mathit{idle}, q \rangle \parallel \langle \llbracket e \rrbracket_\sigma \rangle_f} \\
\\
\text{(READ-FUT)} \\
\frac{f = \llbracket e \rrbracket_\sigma}{o\langle a, \langle l \mid x = e. \mathbf{get}; s \rangle, q \rangle \parallel \langle v \rangle_f \rightarrow o\langle a, \langle l \mid x = v; s \rangle, q \rangle \parallel \langle v \rangle_f} \\
\\
\text{(AWAIT}_1\text{)} \\
\frac{\llbracket e \rrbracket_\sigma = f}{o\langle a, \langle l \mid \mathbf{await} \ e; s \rangle, q \rangle \parallel \langle v \rangle_f \rightarrow o\langle a, \langle l \mid s \rangle, q \rangle \parallel \langle v \rangle_f} \\
\\
\text{(AWAIT}_2\text{)} \\
\frac{\llbracket e \rrbracket_\sigma = f}{o\langle a, \langle l \mid \mathbf{await} \ e; s \rangle, q \rangle \parallel \langle \perp \rangle_f \rightarrow o\langle a, \langle l \mid \mathbf{suspend}; \mathbf{await} \ e; s \rangle, q \rangle \parallel \langle \perp \rangle_f} \\
\\
\text{(SELF-SYNC-CALL)} \\
\frac{l(\mathit{destiny}) = f' \quad \llbracket e \rrbracket_\sigma = o \quad \llbracket \bar{e} \rrbracket_\sigma = \bar{v} \quad \mathit{fresh}(f) \quad \mathit{bind}(o, m, \bar{v}, f) = \langle l' \mid s' \rangle}{o\langle a, \langle l \mid x = e.m(\bar{e}); s \rangle, q \rangle \rightarrow o\langle a, \langle l' \mid s'; \mathit{cont}(f') \rangle, \langle l \mid x = f. \mathbf{get}; s \rangle :: q \rangle \parallel \langle \perp \rangle_f} \\
\\
\text{(SELF-SYNC-RETURN-SCHED)} \\
\frac{l'(\mathit{destiny}) = f}{o\langle a, \langle l \mid \mathit{cont}(f) \rangle, \langle l' \mid s \rangle :: q \rangle \rightarrow o\langle a, \langle l' \mid s \rangle, q \rangle}
\end{array}$$

Figure 2: Operational semantics

identifier of a resolved future, behaves like a skip (cf. rule  $\text{AWAIT}_1$ ). An await on a list of futures is equivalent to a list of awaits for individual futures. If the future corresponding to the guard expression has not been resolved, a **suspend** statement is introduced to enable scheduling another process (cf. rule  $\text{AWAIT}_2$ ).

For synchronous self-calls, possession of the object directly transfers control from the current process to the process of the invoked method and back, bypassing the  $\text{SUSPEND}$  and  $\text{ACTIVATE}$  rules. Technically, a special *cont* instruction is here inserted at the end of the statement list of the new process in rule  $\text{SELF-SYNC-CALL}$ , which is then used to re-activate the caller process in rule  $\text{SELF-SYNC-RETURN-SCHED}$ . The expression *destiny* records the method's future; i.e., *destiny* stores the return address of the method activation.

```

1 class Service(Int limit) {
2   Producer prod = new Producer(); Proxy proxy = new Proxy(limit,this,prod);
3   Proxy lastProxy = proxy;
4
5   Void run() { this!produce(); }
6   Void subscribe(Client cl){Fut<Proxy> f; f = lastProxy!add(cl); lastProxy = f.get;}
7   Void produce(){proxy!start_publish(); }
8 }
9
10 class Proxy(Int limit, Service server, Producer prod) {
11   List<Client> myClients = Nil; Proxy nextProxy;
12
13   Proxy add(Client cl){ Proxy lastProxy = this; Fut<Proxy> f';
14     if length(myClients) < limit {myClients = append(myClients, cl);}
15     else {if nextProxy == null {nextProxy = new Proxy(limit,server,prod);}
16       f' = nextProxy!add(cl); lastProxy = f'.get;} return lastProxy; }
17
18   Void start_publish(){ Fut<Proxy> f''; f'' = prod!detectNews(); await f''?;
19                       News ns = f''.get; this!publish(ns); }
20
21   Void publish(News ns){ myClients!signal(ns);
22     if nextProxy == null {server!produce();} else {nextProxy!publish(ns);} }
23 }

```

Figure 3: Implementation of the publisher-subscriber example.

### 2.3. ABS Example

We provide a publisher-subscriber example in Fig. 3 to present the ABS language. `Service` objects publish news updates to subscribing clients through a chain of `Proxy` objects. Each proxy object handles a bounded number of clients. `Service` objects handle a subscribe request efficiently by delegating its time-consuming parts to `Proxy` objects, and the proxies publish news to clients using asynchronous calls (without futures) to make the cooperation efficient. The code in line 6 expresses that a `Service` object invokes method `add` on a `Proxy` object through method `subscribe`. Similarly, the code in line 16 expresses that a `Proxy` object invokes method `add` on the next `Proxy` object through method `add`. Note that both cases contain asynchronous blocking calls of the form  $f = e!m(\bar{e}); \dots; x = f.\mathbf{get}$ , where there are no suspension (scheduling) points in between. The caller of the `add` method is blocked until it receives the results. On the other hand, a `Proxy` publishes news only when it receives the news. When there is no arrival of news, the `Proxy` object meanwhile performs other actions. This interleaving behaviour is achieved by the usage of **await** statement for explicit process synchronization.

## 3. Colored Petri Nets

Petri nets capture true concurrency in terms of causality and synchronization [14, 15], and consist of places, transitions and arcs. Colored Petri nets (CPNs), a well-established form of high-



level Petri nets [20], extend the basic Petri net formalism to enable the modelling of data [13, 21]. In CPNs, the data values are multisorted or *typed*. Types, representing sets of values, are called *color sets* in CPN terminology, and individual values are seen as colors. A type can be arbitrarily complex, defined by many sorted algebra in the same way as abstract data types. Each place in a CPN is typed; i.e., a place has an associated color set which determines the kind of data (“colored tokens”) the place can contain. Tokens in a typed place represent individual values of that type. In the following, we formally introduce CPNs in their basic form, without hierarchies. Hierarchies, which enrich the nets with modularity for practical reasons, require a more complicated though equivalent definition. The basic definition of CPNs suffices for our purposes as any hierarchical CPN can be unfolded to a semantically equivalent non-hierarchical CPN [13].

**Definition 1** (Colored Petri net). *A colored Petri net (CPN) is a tuple  $(P, T, A, \Sigma, V, C, G, E, I)$  where*

1. *places  $P$  and transitions  $T$  are disjoint finite sets;*
2.  *$A$  is the set of arcs, such that  $A \subseteq (P \times T) \dot{\cup} (T \times P)$ ;*
3.  *$\Sigma$  is a finite set of types (where each individual type is seen as a non-empty “color set”);*
4. *typed variables  $V$  form a finite set, i.e.,  $\text{type}(v) \in \Sigma$  for all  $v \in V$ ;*
5. *a coloring  $C : P \rightarrow \Sigma$  associates a type to each place.*
6. *labeling functions  $G : T \rightarrow \text{Expr}_V$  and  $E : A \rightarrow \text{Expr}_V$  associate expressions with free variables from  $V$  to transitions, resp. to arcs. Expressions associated with transitions are called guards, and we write  $e$  and  $g$  for expressions resp. guards;*
7. *an initialization function  $I : P \rightarrow \text{Expr}_\emptyset$  associates an expression without free variables to every place.*

The “static” structure of a Petri net forms a graph, with the places and transitions as nodes and the arcs as edges. Places and transitions are disjoint by the arc condition from Definition 1(2), so the places, transitions, and arcs of a Petri net form a directed, bi-partite graph: An arc  $(p, t)$  is outgoing for a place  $p$  and incoming for a transition  $t$ , whereas an arc  $(t, p)$  is incoming for  $p$  and outgoing for  $t$ . The guards associated with transitions express synchronization conditions which, together with the expressions on the arcs, capture the transition semantics of CPNs. Since tokens are individually typed values and expressions (including guards) contain free variables, the *enabledness* of transitions depends on the choice of values for the free variables.

We assume that expressions are appropriately typed, as follows. Guards for transitions are Boolean expressions (i.e., for result type of a guard  $g$ ,  $\text{type}(g) = \text{Bool}$ ). Expressions for arcs, on the other hand, are “multi-set typed”: Assume an arc connected to a place  $p$  (either as source or as target of the arc) with  $C(p)$  as the type of  $p$ . Then the expression labelling the arc has multi-sets over  $T$  as resulting type, (i.e.,  $\text{type}(e) = C(p) \rightarrow \mathbb{N}$ ). With the “tokens” of the classical Petri nets now generalized to (appropriately typed) closed expressions, the initialization function attaches a multiset of such closed expressions to each place of the net, such that  $\text{type}(I(p)) = C(p) \rightarrow \mathbb{N}$ , for all places  $p$ . The initialization function corresponds to the so-called initial marking of the net (for the definition of markings, see below).

**Example 2** (Colored Petri nets). *Let’s use the very small net from Figure 4 to illustrate the definition of CPNs and related concepts. The example has two places  $p_1$  and  $p_2$  and one transition  $t$ , connected by two arcs.  $C(p_1)$  and  $C(p_2)$  are respective types from  $\Sigma$  of the two places (attached to the places by the “coloring” function  $C$ ). The two arcs carry expressions  $e_1$  and  $e_2$ , correspondingly,*

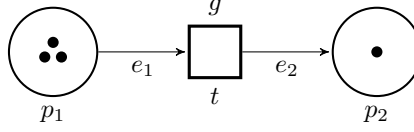


Figure 4: Illustration of CPN

and the transition is decorated by a guard  $g$ . The black “bullets” in the places are tokens as in classical Petri nets; for CPNs, tokens are individual values and each token on  $p_1$  and  $p_2$  must have a data value that belongs to  $C(p_1)$  and  $C(p_2)$ , respectively. See also Example 4 below.

Tokens in a CPN correspond to (values of) closed expressions. The guards on transitions and the expressions attached to the arcs, which together govern the dynamic behavior of a CPN, contain free variables, so their interpretation depends on choosing values for the those variables. Variable bindings (or variable assignments)  $b$  are mappings from variables to values; we assume that bindings respect the types of the involved variables. Let  $Var(t)$  denote the variables of a transition (where  $Var(t) \subseteq V$ ), which are the variables in the guard of  $t$  and the variables of the arc expressions of the arcs connected to  $t$ . The binding of a transition covers all variables from  $Var(t)$ . Let  $\llbracket e \rrbracket_b$  denote the value of expression  $e$  under variable binding  $b$ . A binding satisfies a guard expression  $g$  if  $\llbracket g \rrbracket_b$  evaluates to true.

The current state or configuration of a CPN is given by a so-called *marking* and the dynamic behavior of such a net is described by sequences of *steps*.<sup>2</sup> Markings associate multi-sets of appropriately typed values to places. Steps transform markings in the way specified by the expressions on the arcs and under the condition, that the guards of the concerned transitions are satisfied. Given a CPN, a *marking*  $M$  attaches to each place  $p$  a multiset of appropriately typed values, i.e.  $M(p) : C(p) \rightarrow \mathbb{N}$ . As mentioned shortly earlier, the net’s initialization function  $I$  is used to provide the *initial marking*:  $M_0(p) = \llbracket I(p) \rrbracket$ . Note that the expressions in  $I(p)$  don’t contain variables, hence  $\llbracket I(p) \rrbracket$  is well-defined without a variable binding (resp. under the empty variable binding).

A *step* is a selection of a subset of the net’s transitions together with appropriate bindings for the variables of these transition such that the selected transitions are enabled, as defined below. Technically, a *step*  $Y$  is a function of type  $Y : T \rightarrow ((Var \rightarrow Val) \rightarrow \mathbb{N})$ , assigning (“selecting”) a multi-set of bindings to each transition, where for all transitions  $t$  and all bindings  $b$  from  $Y(t)$ ,  $\llbracket G(t) \rrbracket_b = true$ . As usual, the bindings are assumed to be consistent with the typing and additionally, must cover the variables of the transition. It’s required that the step is non-empty in that for at least one transition, the multi-set of bindings is non-empty (an empty step  $Y$  would correspond to a stutter-transition without effect). Note that the notion of step is defined *independent* from a marking, i.e., independent from the current configuration of a net. Steps can be seen equivalently as a multi-set of pairs of transitions and bindings, i.e., of type  $(T \times (Var \rightarrow Val)) \rightarrow \mathbb{N}$ . We call the elements  $(t, b)$  of such multi-sets also *binding elements*.

To use a *step* to transform a marking into a successor marking, not only must the selected bindings satisfy the guards of the selected transitions. In addition, the marking prior to doing the step must assign “enough” tokens (i.e., values) to the places feeding into to the selected transitions.

<sup>2</sup>One also finds the words “token distribution” and (non-empty) “binding distribution” as alternative terminology for markings and steps.

This is captured by expressions  $E(p, t)$  on the arcs of the selected transitions.

**Definition 3** (Enabledness of transitions and steps). *Let  $\subseteq_m$  denote the ordering relation over multisets. A transition  $t$  is enabled for binding  $b$  in a marking  $M$  if*

1.  $\llbracket G(t) \rrbracket_b = \text{true}$ , and
2.  $M(p) \supseteq_m \llbracket E(p, t) \rrbracket_b$ , for all places  $p \in P$ .

A step  $Y$  is enabled in a marking  $M$  if  $M(p) \supseteq_m \llbracket E(p, t) \rrbracket_Y$  for all places  $p$ , where  $\llbracket E(p, t) \rrbracket_Y$  represents the multi-set  $\sum_{(t,b) \in Y} \llbracket E(p, t) \rrbracket_b$ .

In abuse of notation, we use the multi-set comparison symbols  $\subseteq_m, \supseteq_m$ , etc. also for comparing markings in a pointwise manner, i.e.,  $M_1 \subseteq_m M_2$  iff  $M_1(p) \subseteq_m M_2(p)$ , for all places  $p$ . Note that the condition  $\llbracket G(t) \rrbracket_b = \text{true}$  for guard satisfaction is not needed for the enabledness of steps, as the notion of enabling of a binding already requires that. When  $t$  is enabled for  $b$  in  $M$ , the transition may *occur* or “fire” (given  $b$ , leading to the marking  $M'$  where  $M'(p) = (M(p) - \llbracket E(p, t) \rrbracket_b) + \llbracket E(t, p) \rrbracket_b$ , for all places  $p$ , and where  $+$  and  $-$  on the multiplicity of the elements correspond to multi-set union and multi-set difference (where for the latter  $S_1 - S_2$  is defined only if  $S_2 \subseteq_m S_1$ , which is assured by the condition on steps for being enabled). Similarly for enabled steps  $Y$ ,  $M_1 \xrightarrow{Y} M_2$  denotes that a marking  $M_1$  evolves into  $M_2$  by “firing” step  $Y$ . A (finite) *occurrence sequence* is a sequence of markings and steps of the form

$$M_1 \xrightarrow{Y_1} M_2 \xrightarrow{Y_2} M_3 \dots M_n \xrightarrow{Y_n} M_{n+1} . \quad (1)$$

We also need to refer to the projection of an occurrence sequence onto the involved transitions, i.e., omitting bindings and markings: an *occurrence word* is a finite sequence of transitions, such that there exist bindings and markings such that it gives rise to an occurrence sequence as given in equation (1).

The “true concurrency” semantics typical for Petri nets allows the simultaneous firing of transitions in a step. Whereas steps are required to be non-empty, a step which only fires one pair of transition  $t$  and binding  $b$ , is denoted  $\xrightarrow{t,b}$ . A step semantics restricted to such single transition steps is equivalent to the unrestricted semantics, but corresponds to “interleaving concurrency”. In the rest of the paper we consider such single transition steps and, when obvious, the binding  $b$  is omitted.

**Example 4** (Bindings, markings, steps). *Let’s use Figure 5 to illustrate the introduced concepts. Assume place  $p_1$  on the upper left is typed by a type  $C = \{a, b, c\}$ , and the other two places,  $p_2$  and  $p_3$  are typed by the natural numbers. As for the expressions on the arcs, assume the following: let variables  $x$  and  $y$  be typed by natural numbers and  $z_1$  and  $z_2$  by the three-element type  $C$ .*

$$\begin{aligned} e_2 &= 2 \times_m y \\ e_3 &= 3 \times_m (4x) \\ g &= z_1 \neq z_2 \wedge x \geq y \end{aligned}$$

Multi-set “multiplicity” is stated by the natural number left to  $\times_m$ . The arc from  $p_1$  to  $t$  is not decorated by an expression  $e_1$ , which is assumed as  $e_1 = 0 \times_m z$ .

Assume further the following two bindings

$$b_1 = [x \mapsto 2, y \mapsto 1, z_1 \mapsto a, z_2 \mapsto b] \quad \text{and} \quad b_2 = [x \mapsto 3, y \mapsto 2, z_1 \mapsto a, z_2 \mapsto b] .$$

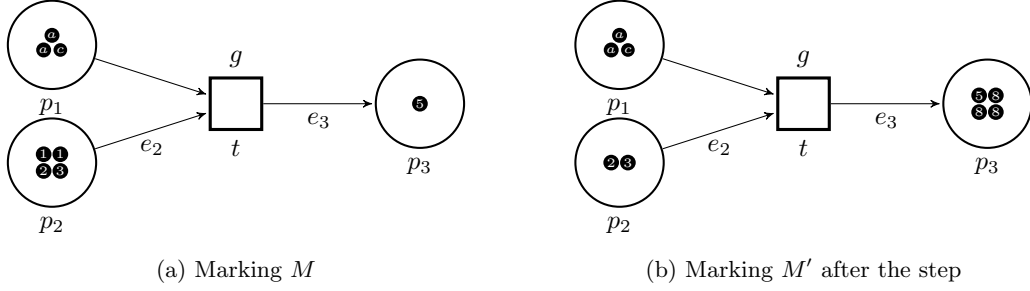


Figure 5: Illustration of markings, bindings, and steps

Both bindings satisfy the guard  $g$ , i.e.,  $\llbracket g \rrbracket_{b_1} = \text{true}$  as well as  $\llbracket g \rrbracket_{b_2} = \text{true}$ . For the marking  $M$  given in Figure 5a, though, only  $b_1$  enables the transition  $t$ . This is due to the arc  $(p_2, t)$ , whose expression  $e_2$  requires a multiplicity of 2 or more for any choice of  $y$ , and in the current marking, that requires  $y \mapsto 1$ . Arc  $(p_1, t)$  does not impose any restrictions on the presence of values in the marking for  $p_1$ . Taking  $(t, b_1)$  as a step, we get  $M \xrightarrow{t, b_1} M'$ , with  $M'$  given in Figure 5b on the right.

#### 4. Translating ABS Semantics to Colored Petri Nets

In this section, we define the translation from ABS to CPNs. After a short introduction covering the core ideas of the translation (Section 4.1), we proceed into a more in depth presentation of how the translation has been implemented. Sections 4.2 and 4.3 highlight crucial parts of how the ABS semantics are represented on the Petri net level, focusing on the active objects creation and the communication mechanism respectively and, in the latter case, we see asynchronous method calls and the resolution of futures via **get** statement.

##### 4.1. Overview over the Petri Net Semantics for ABS

The starting point of the translation are abstract ABS programs, i.e. programs where *data* values have been abstracted already. Remember that, among other questions, reachability in Petri nets is decidable (see [22] for a survey of decidability issues for (classical) Petri nets) but then again, whether data-abstracted ABS programs are Turing complete or not, is an open question. Still, there are two remaining sources of *infinity*: creation of (active) objects and creation of processes and accompanying future references via asynchronous method calls. It should be noted that in absence of synchronous, reentrant method calls, unboundedly growing stacks do not contribute to the potential unboundedness of the state space. In the translation, one can conceptually distinguish between *language-specific* aspects and *program-specific* aspects: the ABS language and its semantics is represented by one CPN, common for all programs. This CPN therefore can be seen as a translation of the ABS-language as such. Roughly, each semantic rule from the operational semantics of Fig. 2 is represented by transitions and places, with appropriate types and guards.

As a result, one particular program, resp. a particular run-time configuration of a program, is represented by a *marking* of the Petri net. The expressive power of *colored* Petri nets is crucial to achieve such a conceptually clear and structural translation: since tokens are distinguishable, the transitions and places operates on typed values making it possible to represent the components of a

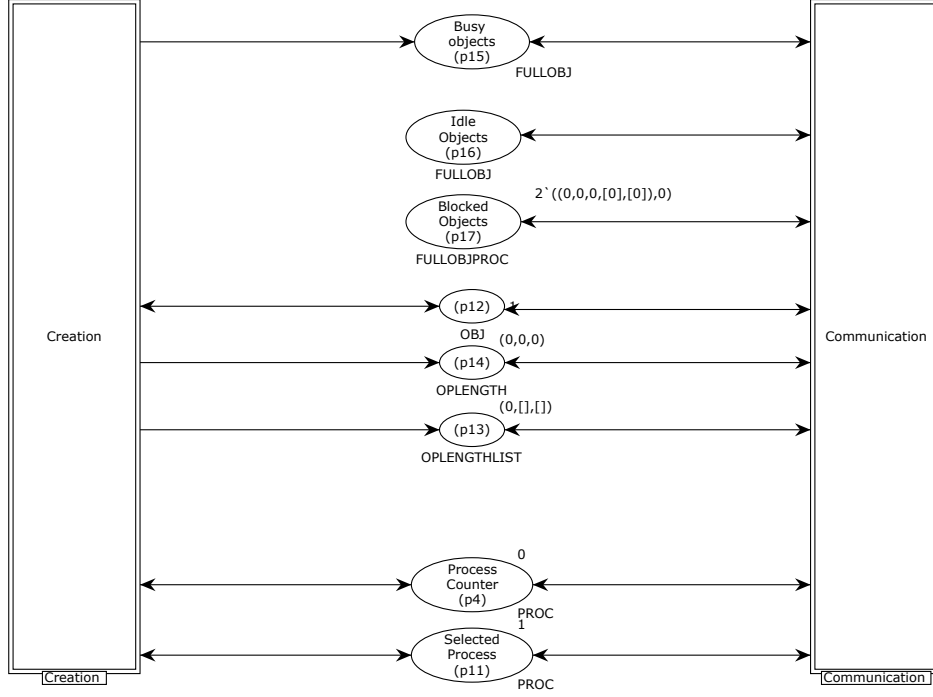


Figure 6: Top-level module of the ABS semantics implementation in CPN Tools

configuration in a clean manner. For instance, object identities and process resp. future identities are represented naturally by resp. types, which correspond to integers.

Here, we should notice that for the implementation of the translation, we used hierarchical CPNs while, in Definition 1 of Section 3 we introduced CPNs without hierarchies. This was a decision made for the sake of the simplicity at the level of the definition. Notice that hierarchical CPNs can be reduced to non-hierarchical ones through unfolding, hence the two definitions are equivalent. At the practical level, hierarchies add more convenience, since they allow organizing the model into smaller parts (modules). This offers better human understanding when the size of the model is large and reusability, if some parts of the net are needed repeatedly. Modules can have a hierarchical structure, hence modules can have submodules. Submodules are hidden parts of the net, appearing in the upper module as a black-box in the form of a so-called *substitution transition*. In the figures, the substitution transitions are the transitions with the double outer lines. Similarly, the places that have double lines are the places that are common in more than one module. The indications in the little rectangular tags next to those places demonstrate their role regarding to each module, i.e. if their marking behave like an input, output or both to the module.

From now on, we will refer to the whole of our implementation of ABS semantics in CPN Tools as CPN-ABS. Figure 6 shows the top-level module of CPN-ABS. It consists of two substitution transitions, one related to the object creation and one related to the communication mechanism of ABS. As we shall see in the following sections, the representation of active objects in the net

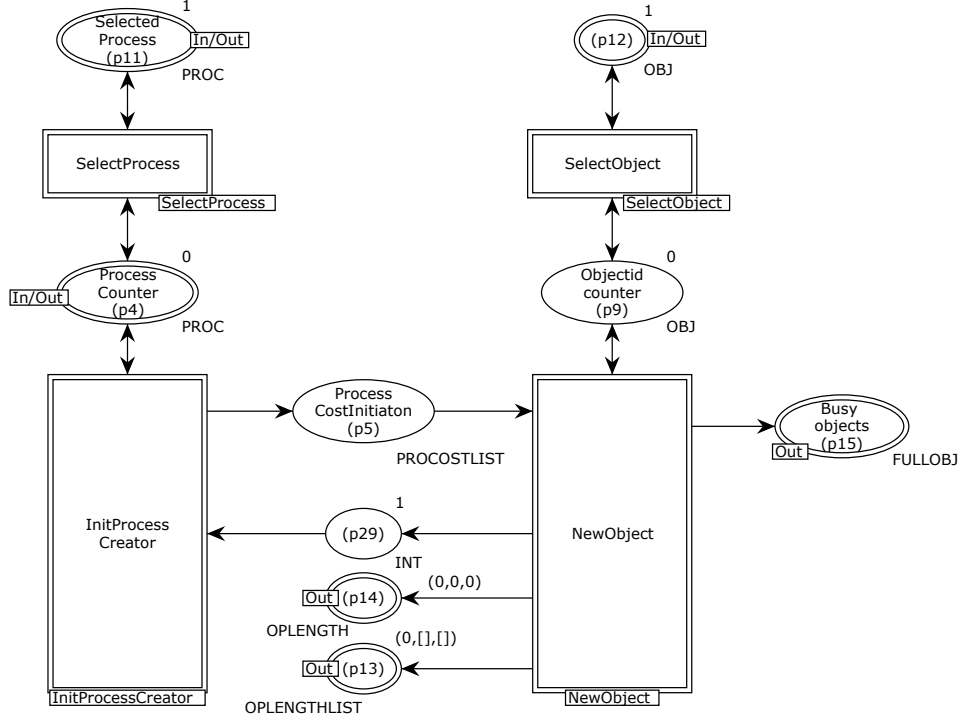


Figure 7: CPN-ABS module for dynamic object creation (i)

is made by tokens carrying the appropriate information. Their creation is related to the substitution transition “*Creation*”. Following the semantics of ABS, they can be located in exactly one among the places “*Busy Objects*”, “*Idle Objects*”, and “*Blocked Objects*”, which are connected to the substitution transition “*Communication*”.

Recall from Section 2 that the communication between objects is achieved through method calls. The rest of the places in Fig. 6 support the soundness of the model keeping information related to the processes (i.e. they contain tokens acting as identifiers or achieving firing order of some transitions). In the implementation, we named most of the transitions and the places in a way reflecting their role to the ABS semantics. We also used a more formal (indexed) naming:  $p_i$  for the places and  $a_j$  for the transitions. This naming will be used mostly in Section 6. In the rest of this section, for the sake of simplicity, we omit referring to details like places, arcs and inscriptions which have an indirect relation with the semantics or with an obvious meaning.

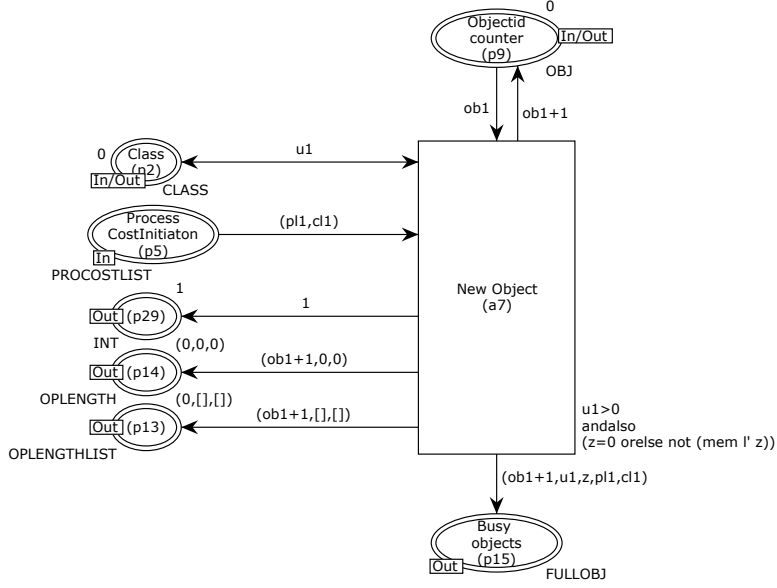


Figure 8: CPN-ABS module for dynamic object creation (iii)

#### 4.2. Dynamic Creation of Active Objects

In this section, we describe the modules that are responsible for the dynamic object creation and conform to the ABS active object creation semantics, hence they are submodules of the top-level module shown in Figure 6 and related to the substitution transition “*Creation*”. In Fig. 7 we see such a module. Transition “*New Object*” substitutes the subnet, which produces tokens representing the ABS objects at the “*Busy Objects*” place. According to the language, each time an object is created, an initial (active) process is dedicated to it. As a result, a fresh process should be created. Same in CPN-ABS, the subnet related to the substitution transition “*InitProcessCreator*” produces this process and inserts it in the (initially empty) process list ( $p_5$ ), which is analog to the process pool each ABS object has.

Figure 8 shows in more detail objects creation. We can see that place  $p_9$  is responsible for the freshness of each new object identifier since its marking is an object counter. Every time the counter increases (var  $ob1 \mapsto ob1 + 1$ ), the fresh value is being passed to the new object tuple in place “*Busy Objects*” ( $p_{15}$ ). This tuple also contains information about the class of the object (var  $u1$ ), taken from place  $p_2$  and the process list (var  $pl1$ ) from  $p_5$  (as explained above). For more details about the process creation, the class creation module and the variable bindings, the interested reader could see Fig. A.15, A.17, A.18 and A.19 of the Appendix.

Remark here that in the object tuples there are some “extra” variables ( $z$  and  $cl1$ ). In our current work they play no active role but their existence is a basis for further development of new modules that add extra features of the language and are worth to analyse in the future, like for example the cost of the programs or the modelling of some abstract execution locations (deployment components).





the **ACTIVATE** rule of ABS, a process from the pool is activated and the inverse with the **SUSPEND** rule. This is simulated by the module related to the substitution transition “*Suspend-Activate*” where, the object is being “moved” from the “*Idle Objects*” place to the “*Busy Objects*” one and vice versa. For more details about the simulation of the process suspension and activation as well as for how the synchronous reentrant self calls have been implemented in CPN-ABS, the reader can see the the appendix (Fig. A.20 and A.21 respectively).

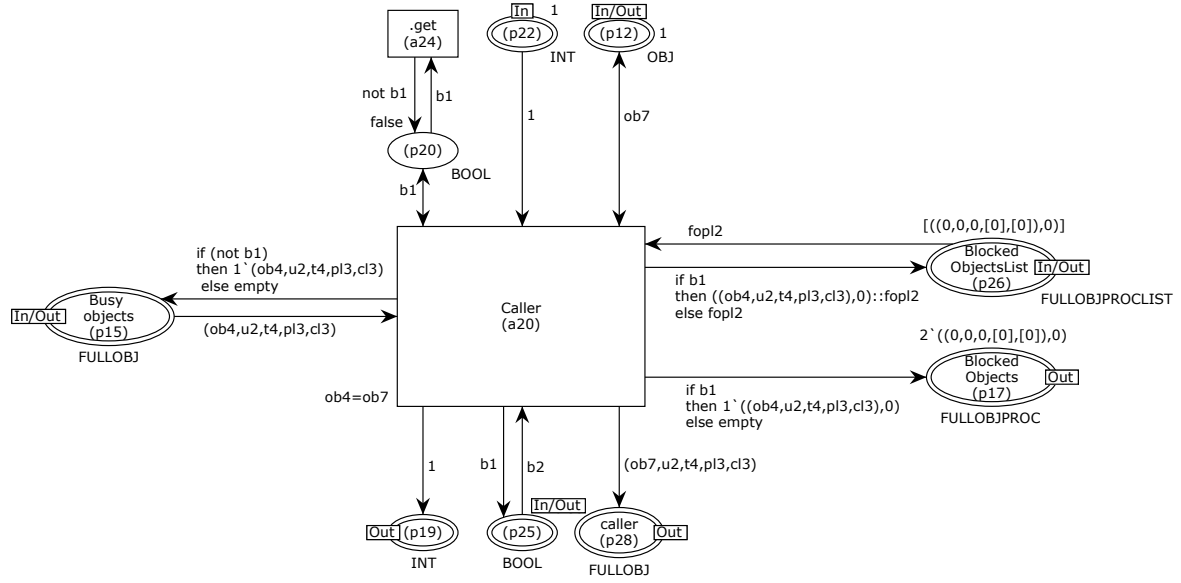


Figure 10: Module for a method caller

Communication between objects involves two parts: the caller and the callee. The module of the Fig. 10 is the part of the net that deals with the caller. Transition “*Caller*” selects the calling object from the “*Busy Objects*” place. There are two cases of communication through asynchronous method calls: immediately followed by a **get** statement or not. Both are simulated by firing the transition “*get*”. It is connected to the place  $p_{20}$  which is of colorset *Bool*. The color of its token is related to the presence of the **get** statement in the obvious way. By firing the transition “*get*”, we alternate the value of the token. So, transition “*Caller*” takes the information on whether the asynchronous call is followed by a **get** statement or not. In the latter case, i.e. when the value of  $b_1$  is false, the transition “*Caller*” maintains the object in the “*Busy Objects*” place following the **ASYNC-CALL** rule of the semantics, otherwise it sends the caller object to the “*Blocked Objects*” place until the waiting future to be resolved (more details about how the future resolution is simulated in CPN-ABS can be found in the appendix)

In the module of Fig. 11 we can see the details about the communication concerning the callee object. As the places related to the status of an object are disjoint, the callee object can reside only in one of the three corresponding places. Therefore, one among the transitions “*Active callee*”, “*Blocked callee*”, and “*Idle callee*” can fire each time for the selected object. Recall that in CPN-ABS, the process pool is implemented as a FIFO queue. As a result, all three of the transitions that refer to the callee object, update its process queue by adding at the end of the list a

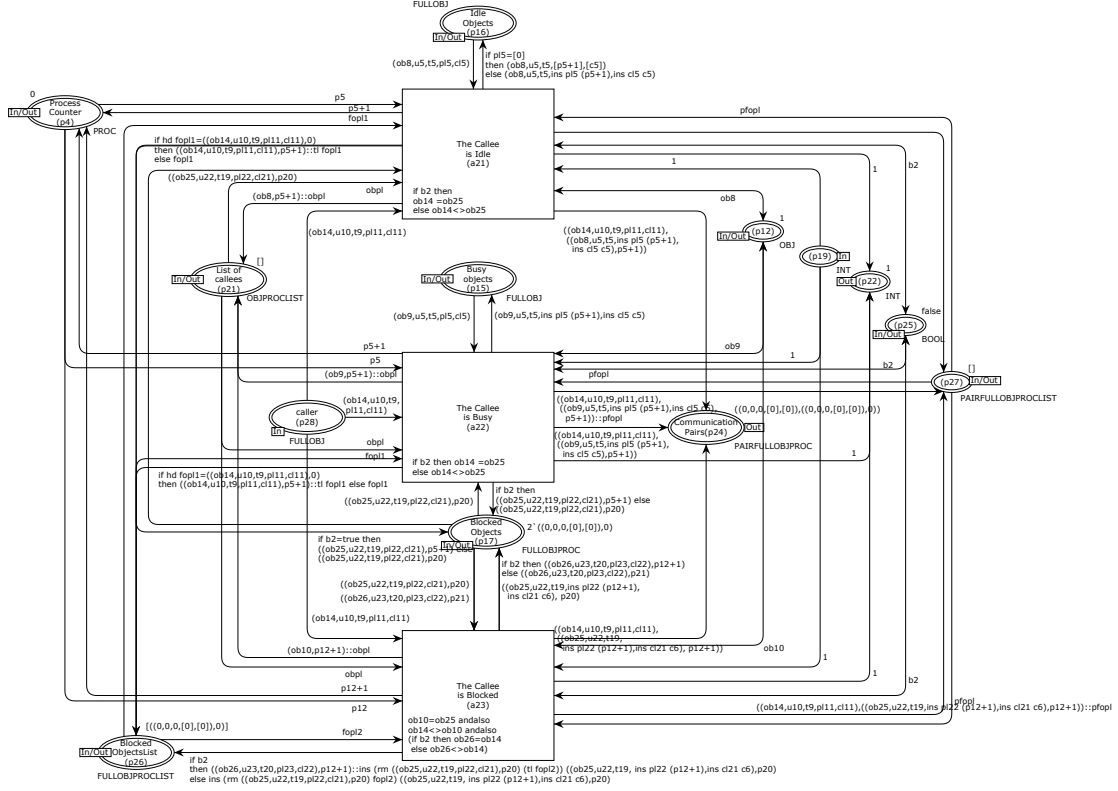


Figure 11: Module for a method callee

new process, related to this particular method call. They also create a communication pair token at the “*Communication pairs*” place by matching the token of the “*Caller*” place (created by the “*Caller*” transition) with the callee object and the process created for this method execution. The marking of this place provides the communication history of the model.

## 5. The Abstraction Function

In this section, we define a translation from ABS configurations to CPN-ABS markings. In its core, it is a structural translation of ABS configurations, ignoring the *data* parts of the program, i.e., the value of variables in the instance states and local states. Hence the translation yields an *abstraction* at the same time, and the resulting CPN-ABS marking over-approximates the original behavior, due to this form of data abstraction.

The translation is given in the form of an abstraction function  $\alpha$ . Remark that in ABS, a configuration is a multiset of objects, invocation messages and futures [12] and each object contains its identifier, an active process and a process pool. In the following, we will define the abstraction

function which selects all this information from a configuration and maps it at the abstract level in the form of CPN–ABS tokens. Then, in Section 6, we will prove that those markings abstractly simulate CPN–ABS program behaviors.

Let  $\mathcal{C}$  be the set of the configurations of an ABS program. Let also  $Obj$  be the set of the objects,  $Class$  the set of its classes,  $Proc$  the set of the processes,  $Msg$  the set of the method invocation messages and  $F$  the set of the resolved futures. Then, we define the functions that project the above sets from ABS configurations as follows:

- $ob : \mathcal{C} \rightarrow Obj$  which projects the objects in an ABS configuration,
- $cl : Obj \rightarrow Class$  which projects object classes of an ABS configuration,
- $pr : Obj \rightarrow \mathcal{P}(Proc)$  which projects the process pools of the objects of ABS configurations,
- $msg : \mathcal{C} \rightarrow Msg$  which projects the messages  $Msg$  of an ABS configuration and
- $fut : \mathcal{C} \rightarrow F$  which projects the set of resolved futures that are related to **get** statements for a configuration.

We define the following injections from the above sets to the set of the positive integers:  $h : Obj \rightarrow \mathbb{Z}^+$ ,  $d : Class \rightarrow \mathbb{Z}^+$ , and  $g : Proc \rightarrow \mathbb{Z}^+$ . Function  $h$  is an injection from the set of objects  $Obj$  of a ABS program to the set of positive integers representing the object identifiers, whereas  $d$  and  $g$  returns the unique identifiers of classes and processes, respectively. Then, let  $m : Msg \rightarrow Proc$  be the injection which maps each invocation message to the process that will be created for the execution of the called method. Let furthermore  $fr : F \rightarrow Proc$  be the injection from the set of resolved futures  $F$  related to **get** statements, to the set of processes  $Proc$ . Finally, let  $pq : \mathcal{P}(Proc) \rightarrow \mathcal{P}(\mathbb{Z}^+)$  be the mapping from the process pools to sets of (unique) positive integers such that for every process pool  $S$ ,  $pq(S) = \{g(s) \in \mathbb{Z}^+ \mid s \in S\}$ .

In CPN–ABS, we model objects as tokens which carry information about their identity, their class and their process pool. As a consequence, each object is represented as a triple  $(id, class, q)$ , where  $id$  is the object identifier of type  $Int$ ,  $class$  is the corresponding class of the object (class identifier) of type  $Int$  and  $q$  is the process pool of the object of type *list of integers*  $LInt$ . Those object-tokens can be located in places corresponding the particular status of the object (idle, active or blocked). CPN–ABS also supports other useful information taken from the configurations which are necessary for the communication between the objects, as for example which process has been created after a method invocation or which process corresponds to a resolved future related to a **get** statement. In both cases, processes are represented as tokens of type  $Int$  but this information comes from different parts of the concrete configuration (messages and futures), hence we use different functions for its extraction.

Now, we can define the abstraction function  $\alpha$ , mapping ABS configurations to CPN–ABS tokens carrying the information discussed above. In the following,  $P$  is the set of the places and  $M(p)$  is the marking of a place  $p$  in CPN–ABS. Then, for all configurations  $c \in \mathcal{C}$ :

$$\begin{aligned} \alpha(c) = \bigcap \{ & M \mid \exists p, p', p'' \in P \text{ s.t. } p \neq p' \neq p'' \text{ and for all } ob(c) \in Obj, \\ & ((h \circ ob)(c), (d \circ cl \circ ob)(c), (pq \circ pr \circ ob)(c)) \in M(p) \\ & \wedge (m \circ msg)(c) \in M(p') \\ & \wedge (g \circ fr \circ fut)(c) \in M(p'') \} , \end{aligned} \quad (2)$$

where,  $\bigcap$  denotes intersection over sets of multisets. With the above equation we define the abstraction of an ABS configuration as the intersection of the CPN–ABS markings containing (i) the

objects (second line of equation 2), (ii) the invocation messages of the method calls, if any (third line of equation 2) and (iii) the resolved future from a method call containing a **get** statement (fourth line of equation 2). We used composition of the functions defined earlier in this section to obtain the appropriate color of the CPN–ABS tokens, starting from ABS (concrete) configurations. The existential quantifiers for the places mean that the above tokens can be located to different places depending every time on the configuration. Observe that, for every ABS configuration, the above intersection is nonempty, i.e. there is a marking such that all the objects of the configuration are represented as tokens in specific places of the model. As we will prove in the next section, CPN–ABS *abstractly simulates* ABS programs. As a consequence, there exist “extra” markings in CPN–ABS which are not important at the level of the abstraction though they are structurally important for the model.

## 6. Soundness Proof of the Translation

This section proves the the soundness of the translation. Since the translation from Section 4 involves abstraction on data, the result of the translation over-approximates the behavior of the ABS program and the soundness of the construction is proven in a standard manner by a *simulation* relation between the small step operational semantics of ABS and the transitions of CPN–ABS.

In particular, we prove that, for any ABS configuration  $c$ , if  $c \rightarrow c'$ , then there exists a marking  $M'$  and an occurrence *sequence*  $\alpha(c) \rightarrow^* M'$  such that the diagram from Figure 12 commutes.

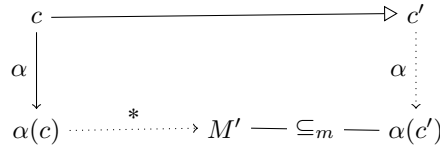


Figure 12: Abstract (weak) simulation relation between ABS configurations and CPN–ABS markings

The ABS semantics as such will *structurally* be translated into one global CPN, but the dynamic behavior executing an individual rule gives rise to a finite sequence of steps in the resulting CPN, as depicted in the simulation of Figure 12. Remember from equation (1), that an occurrence sequence is a sequence of markings and steps where we will focus on singleton steps of the form  $(t, b)$ , with  $t$  being a transition and  $b$  a binding. In the translation, the run-time information, i.e., the bindings and the markings, are not in the picture yet. Omitting that dynamic information from an occurrence sequence, we called such a sequence an *occurrence word*.

So, each transition step from  $c$  to  $c'$  is justified by one rule of the operational semantics, and such steps are thereby translated into a sequence  $\vec{t} = t_1 t_2 \dots t_k$  of *transitions* from  $T$  of the given net. To establish the simulation relation therefore means to prove that the transitions from  $\vec{t}$  can in fact fire in the order given by the translation, in other words that  $\vec{t}$  is an occurrence *word*. For occurrence sequences, remember the definition from equation (1) and that we focus here on “single transition” steps.

In the following we give this proof in detail. After some preliminary definitions on colored Petri nets in Section 6.1, we continue in Section 6.2 mapping each semantic rule of ABS to a sequence of

CPN–ABS transitions. For that mapping, we prove that these sequences correspond to occurrence sequences, thereby establishing the relation of Fig. 12 (see Theorem 12).

### 6.1. Preliminaries

We start with some definitions and lemmas to achieve modularity for the construction. Let's call a transition enabled in a marking  $M$ , if there exists a binding  $b$  s.t.,  $(t, b)$  is enabled in  $M$ . Similarly, we write  $M \xrightarrow{t} M'$  if  $M \xrightarrow{(t, b)} M'$  for some  $b$ . Let  $En(M)$  represent the set of *enabled transitions* for a given marking  $M$ , and  $\mathcal{M}_{reach}$  the set of reachable markings of a net.

In the definition of the translation and the subsequent proof, we often refer to the figures showing the corresponding parts of the CPN, i.e., Figures 6 – 11 from Section 4, resp. Figures A.15 – A.23 from AppendixA. The transitions in the figures are identified by labels  $a_1 \dots a_{25}$  and the places by  $p_1 \dots p_{29}$ . So, when describing the translation later and in the proof, we use those labels to identify the transitions from the net. We use the transitions and their labels interchangeably, i.e., also speak of a “transition  $a_i$ ” when  $a_i$  is the label as used in the tool. Likewise, we use the notion of occurrence words for sequences of labels, (not just for sequences of transitions) We write  $\epsilon$  for the empty word, for instance, for the empty sequences transitions.

In the proof later, some transitions are always enabled and can fire, when needed. That will be the case typically for transitions capturing a “generative” or “counting” nature.

**Definition 5** (Uniformly enabled transition). *A transition  $t$  is called uniformly enabled if, for any reachable  $M$ ,  $t \in En(M)$ .*

Obviously, a transition enabled in a given (reachable) marking can be taken arbitrarily many times in a row.

**Lemma 6** (Uniformly enabled transitions). *For a uniformly enabled transition  $t$ ,  $t^*$  is an occurrence word.*

*Proof.* Obvious. □

Based on the notion of enabled transitions, we define a transition's successor in an occurrence sequence as follows:

**Definition 7** (Post-transitions). *The post-transitions of a transition  $t \in T$  for a given reachable marking  $M$  is defined as  $PostTrans(t, M) = \{t' \in En(M') \mid M \xrightarrow{t} M'\}$ .*

**Lemma 8** (Composition of occurrence sequences). *The composition of an occurrence sequence  $M_1 \xrightarrow{t_1} M_2 \xrightarrow{t_2} \dots \xrightarrow{t_n} M_{n+1}$  with another occurrence sequence  $M'_1 \xrightarrow{t'_1} M'_2 \xrightarrow{t'_2} \dots \xrightarrow{t'_m} M'_{m+1}$  is the occurrence sequence  $M_1 \xrightarrow{t_1} M_2 \xrightarrow{t_2} \dots \xrightarrow{t_n} M_{n+1} \xrightarrow{t'_1} M'_2 \xrightarrow{t'_2} \dots \xrightarrow{t'_m} M'_{m+1}$ , whenever  $M'_1 \subseteq_m M_{n+1}$  and  $\llbracket G(t'_1) \rrbracket_{b_{n+1}} = \text{true}$  and furthermore  $\bigwedge_{2 \leq i \leq m} \llbracket G(t'_i) \rrbracket_{b_i} = \text{true}$  and  $M'_j \subseteq_m M'_j$ , for all  $2 \leq j \leq m + 1$ .*

*Proof.* For the prefix of the sequence which is identical to the first composed sequence, the result is trivial. Then, since  $M'_1 \subseteq_m M_{n+1}$ , after  $t'_1$ , obviously, if  $\llbracket G(t'_2) \rrbracket_{b'_2} = \text{true}$ , then  $M'_2 \subseteq_m M'_2$ , and so on. □

For composition of occurrence words, we get as immediate corollary:

**Corollary 9** (Composition of occurrence words). *The concatenation of two occurrence words is an occurrence word if the corresponding occurrence sequence is the composition of the occurrence sequences of the concatenated words.*

*Proof.* Trivial, from the labelling function and the composition Lemma 8.  $\square$

## 6.2. Soundness of CPN-ABS

The colored Petri net CPN-ABS representing the ABS semantics has 25 transitions; we assume them labeled  $a_1, a_2, \dots, a_{25}$ . The label that corresponds to each transition appears with the form  $(a_i)$  below the name of the transition on CPN-ABS. Recall that  $M(p)$  is the marking of each individual place  $p$ , while  $M$  is the marking of the total CPN, i.e., the vector  $(M(p_1), \dots, M(p_l))$ , where  $P = \{p_1, \dots, p_l\}$  is the set of places of the CPN.

### 6.2.1. The translation function

The translation maps semantic rules from Figure 2 to sequences of net transitions, here represented by their labels  $a_i$  as shown in the figures. The mapping is shown in overview in Table 1. The translation per rule is fixed insofar the involved transitions and their order is concerned. To establish the simulation relation, some of the transitions have to fire not a fixed number of times, but a variable number, where the number of iterations depends on the current run-time configuration in ABS (resp. the current marking in the CPN when doing the simulation). Instead of writing  $a^*$  for an arbitrary iteration of a transition labelled  $a$ , the mapping from Table 1 indicates that number when needed writing  $a_i^{n_i}$  where  $n_i$  indicates the number of times  $a_i$  needs to fire.

rule	sequence of transitions
SKIP, ASSIGN <sub>i</sub>	$\epsilon$
ACTIVATE	$a_{11}^{n_{11}} a_{12}^{n_{12}} a_{14}^{n_{14}} a_{15}^{n_{15}} a_{18}$
SUSPEND	$a_{11}^{n_{11}} a_{12}^{n_{12}} a_{14}^{n_{14}} a_{15}^{n_{15}} a_{17}$
ASYN-CALL	$a_{14}^{n_{14}} a_{15}^{n_{15}} a_{24}^{n_{24}} a_{20}^{n_{20}} a_{14}'^{n_{14}'} a_{15}'^{n_{15}'} a_{21}^{n_{21}} a_{22}^{n_{22}} a_{23}^{n_{23}}$
RETURN	$a_{11}^{n_{11}} a_{12}^{n_{12}} a_{14}^{n_{14}} a_{15}^{n_{15}} a_{19}$
READ-FUT	$a_{25}$
NEW-OBJECT	$a_1^{n_1} a_2^{n_2} a_3^{n_3} a_6 a_7$
SYNC-SELF-CALL	$a_{14}^{n_{14}} a_{15}^{n_{15}} a_{16}$

Table 1: Mapping from rules to sequences of transitions,

To define these numbers, we make use of the following notation: for two natural numbers  $n$  and  $n'$ , let  $n \dot{-} n'$  denote the non-negative difference, i.e.,  $n \dot{-} n' = n - n'$  if  $n > n'$ , and 0 otherwise. Often, the translation uses two numbers  $m = n \dot{-} n'$  and conversely  $\hat{m} = n' \dot{-} n$  to define the sequence  $a^m \hat{a}^{\hat{m}}$ . In such situations, at least one of  $a^m$  and  $\hat{a}^{\hat{m}}$  equals the empty sequence  $\epsilon$  (due to the law of trichotomy according to which every number is either negative, positive, or else zero).

In the following definitions and the subsequent proof, we also make use of the following notation. In a number of cases, it will be an invariant for a place  $p$  to have exactly one value, i.e., for the multi-set  $M(p)$ ,  $|M(p)| = 1$ . In these cases, we write also  $M(p)$  to refer to that value (as opposed to its multiplicity, which is uniformly 1).

Now, for rules ACTIVATE and SUSPEND (covering also SYNC-SELF-CALL and partially ASYNC-CALL, as well), we set

$$\begin{aligned} n_{11} &= g(p) \div M(p_{11}) \\ n_{12} &= M(p_{11}) \div g(p) \\ n_{14} &= h(o) - M(p_{12}) \\ n_{15} &= M(p_{12}) - h(o) , \end{aligned} \tag{3}$$

where place  $p = \text{select}(q, a)$  according to the ACTIVATE rule.

For rule ASYNC-CALL, we set  $n_{24} = \{0, 1\}$ . while  $n'_{14}$  and  $n'_{15}$  are defined as  $n_{14}$  and  $n_{15}$ , respectively, in the previous case, with the difference that the value of  $h(o)$  for  $n_{14}$  and  $n_{15}$  refers to the caller, while for  $n'_{14}$  and  $n'_{15}$ , it refers to the callee object of the asynchronous method call. In the corresponding semantic rule of *ABS*, those values (i.e. the values of  $h^{-1}$ ) are denoted as  $o$  and  $o'$  respectively for the caller and the callee. Furthermore, for  $a_{21}a_{22}a_{23}$ , the numbers are such that  $n_{21} + n_{22} + n_{23} = 1$ . Remark here that,  $M(p_{15})$ ,  $M(p_{16})$  and  $M(p_{17})$  are pairwise disjoint (in particular  $M(p_{15}) + M(p_{16}) + M(p_{17}) = |\text{Obj}|$  is a place invariant), so, at most one among the transitions  $a_{21}$ ,  $a_{22}$  and  $a_{23}$  can fire. Cf. Figure 11 for the part of the net representing the callee of a method.

For rule NEWOBJECT we set

$$\begin{aligned} n_1 &= d(o) \div M(p_1) \\ n_2 &= d(o) \div M(p_2) \\ n_3 &= M(p_2) \div d(o) . \end{aligned} \tag{4}$$

In the definition,  $d(o)$  indicates (the number representing) the class where object  $o$  belongs to.

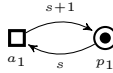
### 6.2.2. Soundness

As the transitions of the target net are uniquely labeled by elements  $a_i$ , we use the labels interchangeably with the transitions, i.e. we write, for example,  $a \in \text{En}(M)$  for  $t \in \text{En}(M)$  etc. We also omit referring to the binding elements where obvious, writing, for example,  $G(a)$  instead of  $\llbracket G(t) \rrbracket_b$  and  $E(p, a)$  instead of  $\llbracket E(p, t) \rrbracket_b$ . We write  $|M(p)|$  for the number of tokens of the place  $p$ , i.e. the cardinality of the multiset  $M(p)$ .

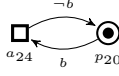
Now, we can proceed on the soundness proof of CPN-ABS.

**Lemma 10** (Uniform enabledness for  $a_1$  and  $a_{24}$ ). *Transitions  $a_1$  and  $a_{24}$  are uniformly enabled.*

*Proof.* For transition  $a_1$  from Figure A.17, the relevant situation, i.e., the involved places and transitions look as follows, the loop being responsible to increment the counter:



The guard of  $p_1$  is uniformly true, and  $a_2$ , the only other transition adjacent to  $p_1$ , leaves any marking of  $p_1$  unchanged (the inscriptions on the incoming and outgoing arcs from  $p_1$  to this transition coincide); no other arcs are adjacent to  $a_1$ . Hence, transition  $a_1$  is uniformly enabled. For transition  $a_{24}$  from Figure 10, the relevant portion of the nets looks similar



toggling the boolean token in  $p_{20}$  (initially being false). Again, the guard of the considered transition is true. Furthermore, transition  $a_{20}$ , also adjacent to  $p_{20}$ , leaves any marking of  $p_{20}$  unchanged, and since no other transition interferes with  $p_{20}$ ,  $a_{24}$  is uniformly enabled, which concludes the argument.  $\square$

The next lemma is the core of the simulation argument, establishing, rule by rule, that the translation simulates the steps of the operational semantics.

**Lemma 11.** *For each rule of the operational semantics from Figure 2, if the rule's hypothesis is satisfied, then the translation of the rule is an occurrence word.*

*Proof.* Proceed by case analysis on the rules of the operational semantics.

*Case:* SKIP, ASSIGN<sub>1</sub>, ASSIGN<sub>2</sub>

These rules are translated to the empty sequence  $\epsilon$  of transitions; hence their cases are immediate.

*Case:* AWAIT<sub>1</sub>, AWAIT<sub>2</sub>

These rules are syntactic sugar for SKIP and SUSPEND; hence they are omitted from the proof.

*Case:* NEW-OBJECT with translation  $a_1^{n_1} a_2^{n_2} a_3^{n_3} a_6 a_7$ .

See Figures A.17 (for  $a_1$ ,  $a_2$ , and  $a_3$ ), Figure A.16 (for  $a_6$ ), and Figure 8 (for  $a_7$ ). For the definition of  $n_1$ ,  $n_2$ , and  $n_3$ , see equation (4). From that definition, either  $n_2$  or  $n_3$  is 0.

We start by establishing that the first three transitions  $a_1^{n_1} a_2^{n_2} a_3^{n_3}$  are an occurrence word. With  $a_1$  being uniformly enabled by Lemma 10, thus, with Lemma 6,  $a_1^{n_1}$  is an occurrence sequence.

After firing  $a_1^{n_1}$  we know  $M(p_1) \geq d(o)$ . More precisely, for  $n_1 \geq 1$ , i.e., firing  $a_1$  at least once, we know  $M(p_1) = d(o)$  after  $a_1^{n_1}$ .

*Subcase:*  $d(o) > M(p_2)$

Consequently  $n_2 = d(o) - M(p_2)$  in this case. Note that the guard of  $a_2$  corresponds to requiring  $M(p_1) < M(p_2)$  as precondition for firing (the guard reads  $u < s$ , where  $u$  and  $s$  are the variables used in the arcs). Furthermore, in this case,  $n_3 = 0$  and  $a^{n_3} = \epsilon$ , and thus  $a_1^{n_1} a_2^{n_2} a_3^{n_3}$  is an occurrence word.

*Subcase:*  $d(o) < M(p_2)$

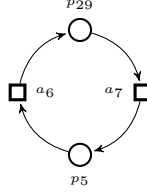
This time  $n_2 = 0$  and  $n_3 = M(p_2) - d(o)$ . The guard of  $a_3$  (stating  $u > 1$ ) evaluates to true for all successive steps of  $a_3$  (each decrementing the corresponding counter in place  $p_2$  by 1). Choosing also  $n_1 = 0$  makes  $a_1^{n_1} a_2^{n_2} a_3^{n_3}$  an occurrence word.

*Subcase:*  $d(o) = M(p_2)$

In this case, we can choose  $n_1 = n_2 = n_3 = 0$ , in which case  $a_1^{n_1} a_2^{n_2} a_3^{n_3}$  is trivially an occurrence word.

Now for subsequent firing of  $a_6$ . In our case,  $M(p_3) = 0$ . Furthermore, for  $p_4$  (representing a counter for processes), we have as invariant  $|M(p_4)| = 1$ : Initially  $|M_0(p_4)| = 1$ , and  $\sum |E(p_4, t)| = \sum |E(t, p_4)|$  where the sums range over the expressions on the arcs adjacent to place  $p_4$  (cf. Figures 6, 7, 9, 11, A.15, A.18, and A.21). So, in order for  $a_6$  to fire, it needs a token from  $p_{29}$  (cf. Figure A.15 and for place  $p_{29}$ , see Figures A.15 and A.16). Transitions  $a_6$  and  $a_7$  are connected as follows





where the picture shows *all* arcs adjacent to  $p_{29}$  and to  $p_5$  (but not all arcs connected to the transitions are reproduced). Transition  $a_6$  can occur, either initially or only after an occurrence of  $a_7$ . Similarly,  $a_7$  can occur only after an occurrence of  $a_6$ .

From the above, we can conclude that, after  $a_1^{n_1} a_2^{n_2} a_3^{n_3}$ ,  $M(p_2) > 0$  and since  $M(p_7) = 0$  (cf. Figure 7), that the guard of  $a_6$  is true. In addition, for  $p_9$  (representing a counter for objects, cf. Figures 7, 8, A.16, and A.19), we have as invariant  $|M(p_9)| = 1$ : Initially  $M_0(p_9) = 1$ , and  $\sum E(p_9, t) = \sum E(t, p_9)$ , where the sums range over the expressions on the arcs adjacent to place  $p_9$ . Thus, also  $a_7$  can occur and the translation  $a_1^{n_1} a_2^{n_2} a_3^{n_3} a_6 a_7$  of NEW-OBJECT is an occurrence sequence, concluding the case.

*Case:* ACTIVATE with translation  $a_{11}^{n_{11}} a_{12}^{n_{12}} a_{14}^{n_{14}} a_{15}^{n_{15}} a_{18}$   
A similar argument as before for  $a_2^{n_2} a_3^{n_3}$  shows that  $a_{11}^{n_{11}} a_{12}^{n_{12}}$  as well as  $a_{14}^{n_{14}} a_{15}^{n_{15}}$  are occurrence sequences, and so is their composition by Lemma 8. From the definition of the function  $g$  in Section 5 and the hypothesis  $p = \text{select}(q, a)$  of rule ACTIVATE, there exists an object s.t.,  $h(o) \in M(p_{16})$ ,  $h(o) \in M(p_{12})$ , and  $g(p) \in M(p_{11})$ . Thus, the guard of  $a_{18}$  is true and  $a_{18}$  can fire. As a result,  $a_{11}^{n_{11}} a_{12}^{n_{12}} a_{14}^{n_{14}} a_{15}^{n_{15}} a_{18}$  is an occurrence word, which concludes the case.

*Case:* SUSPEND with translation  $a_{11}^{n_{11}} a_{12}^{n_{12}} a_{14}^{n_{14}} a_{15}^{n_{15}} a_{17}$

Similar to the previous case.

*Case:* RETURN, with translation  $a_{11}^{n_{11}} a_{12}^{n_{12}} a_{14}^{n_{14}} a_{15}^{n_{15}} a_{19}$

Similar to the previous cases, making use of the invariant  $|M(p_{18})| = |M(p_{14})| = |M(p_{21})| = |M(p_{22})| = 1$ , which is easy to establish, as done in a previous case, for instance for  $|M(p_9)|$ .

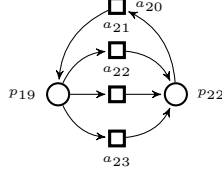
*Case:* READ-FUT with translation  $a_{25}$

Immediate (cf. Figures 9 and especially A.23).

*Case:* ASYNC-CALL and BIND-MTD (taken as a single rule since they occur sequentially) with translation  $a_{14}^{n_{14}} a_{15}^{n_{15}} a_{24}^{n_{24}} a_{20} a_{14}^{n'_{14}} a_{15}^{n'_{15}} a_{21}^{n_{21}} a_{22}^{n_{22}} a_{23}^{n_{23}}$

It's straightforward to establish that  $a_{14}^{n_{14}} a_{15}^{n_{15}}$  and as well as  $a_{14}^{n'_{14}} a_{15}^{n'_{15}}$  are occurrence words. In addition,  $a_{24}$  is a uniformly enabled transition, so its iteration  $a_{24}^{n_{24}}$  is an occurrence word (cf. Lemma 6), and consequently, with Lemma 8, the composition  $a_{14}^{n_{14}} a_{15}^{n_{15}} a_{24}^{n_{24}}$  is an occurrence word, as well.

Places  $p_{22}$  and  $p_{19}$  play similar role for alternating occurrences of  $a_{20}$  and one among  $a_{21}, a_{22}$  and  $a_{23}$  as it was the case for places  $p_{29}$  and  $p_5$  and transitions  $a_6$  and  $a_7$  in the case for NEW-OBJECT above. See Figure 10 for  $a_{20}$  (a transition crucial for dealing with the behavior of the caller), and Figures 9 and 11, as well as Figures A.22 and A.23 for transitions  $a_{21}$ ,  $a_{22}$ , and  $a_{23}$  (covering behavior of a callee), resp. the following:



In addition, we have as invariant  $|M(p_{25})| = 1$ . So,  $a_{20}$  can fire as soon as  $h(o) = M(p_{12})$ , which is the case after  $a_{14}^{n_{14}} a_{15}^{n_{15}} a_{24}^{n_{24}}$ . So, from Lemma 8,  $a_{14}^{n_{14}} a_{15}^{n_{15}} a_{24}^{n_{24}} a_{20}$  is an occurrence word. As stated, also  $a_{14}^{n'_{14}} a_{15}^{n'_{15}}$  is an occurrence word. Now since  $M(p_{15})$ ,  $M(p_{16})$ , and  $M(p_{26})$  are pairwise disjoint, only one among  $n_{21}$ ,  $n_{22}$ , and  $n_{23}$  can be different from 0. In addition we have  $|M(p_{21})| = |M(p_4)| = |M(p_3)| = |M(p_{25})| = |M(p_{25})| = |M(p_{26})| = |M(p)| = 1$ ,  $M(p_{17}) \neq \emptyset$  and  $|M(p_{28})| = 1$  after the occurrence of  $a_{20}$ . So, with Lemma 8, also  $a_{14}^{n'_{14}} a_{15}^{n'_{15}} a_{21}^{n_{21}} a_{22}^{n_{22}} a_{23}^{n_{23}}$  is an occurrence word and then, so is  $a_{14}^{i_{14}} a_{15}^{j_{15}} a_{24}^t a_{20} a_{14}^{i'_{14}} a_{15}^{j'_{15}}$  (again with Lemma 8).

*Case:* SYNC-SELF-CALL and SELF-SYNC-RETURN-SCHED (taken as a single rule since they occur sequentially) with translation  $a_{14}^i a_{15}^j a_{16}$

Similar. □

With the simulation theorem that follows, the soundness proof of the translation ABS programs into colored Petri nets is completed.

**Theorem 12** (Simulation). *CPN-ABS markings are in an abstract (weak) simulation relation with ABS program configurations.*

*Proof.* We need to prove that, for any ABS configuration  $c$ , if  $c \rightarrow_r c'$  for some semantic rule  $r \in \text{Sem}$ , then there exists a marking  $M'$  and an occurrence word given by  $\text{Tr}(r)$ , such that  $\alpha(c) \xrightarrow{\text{Tr}(r)} M'$  and  $\alpha(c') \subseteq_m M'$ . This follows straightforwardly from the definition of the abstraction function  $\alpha$ , the image of  $\text{Tr}$ , and from Lemma 11. □

## 7. Communication Analysis

As we saw in detail in the previous section, CPN-ABS markings abstractly simulate ABS program configurations. By construction, CPN-ABS follows the concurrency of ABS and contains its full communication mechanism (see Section 4). As we have already mentioned in Sections 1 and 4, CPN-ABS was implemented in CPN Tools, a model checker for colored Petri nets. This, together with the abstraction relation described in Sections 5 and 6, allow to CPN-ABS to behave as an abstract interpreter for ABS programs. In particular, it overapproximates the communication topologies of ABS programs upon initialisations arising from the application of the abstraction function to the (static view of the) program. This make CPN-ABS useful for communication analysis for ABS programs. In the rest of this section we will illustrate this by applying deadlock and livelock analysis. In particular, we will express the above notions of concurrency in terms of CPN-ABS and explain how we can use it in order to detect deadlocks and livelocks.

### 7.1. Deadlocks

CPN-ABS contains three disjoint places, where, depending on the status of objects (i.e. active, idle or blocked), objects can be located. The place “Blocked Objects” which hosts the blocked objects has a color set of pair  $(ob, p)$ , where  $ob$  is object invoking an asynchronous call with a get-statement, i.e. an asynchronous blocking call, and  $p$  is the process that has been added to the process queue of the callee for the execution of the called method. Recall that  $ob$  is of color  $(id, class, q)$ , where  $id$  is object identity,  $class$  is the class that the object belongs to, and  $q$  is the process queue of the object.

**Definition 13** (Deadlocks in CPN-ABS). *In CPN-ABS, there is a deadlock cycle [23] if and only if there exists a marking of the place “Blocked Objects”, in which there exists  $n$  tokens  $(ob_1, p_1)$  to  $(ob_n, p_n)$  that form a cycle, i.e. for  $1 \leq i < n$ ,  $p_i \in q_{i+1}$  and  $p_n \in q_1$  (where  $q_i$  is the process queue of the  $i^{th}$  object).*

This deadlock situation can be detected by the state space report of the model checker of the CPN Tool used to implement CPN-ABS.

#### 7.1.1. Deadlock Detection

Communication deadlocks can be detected fully automatically using the model checker of CPN Tools to construct the state space of the CPN model for a given ABS program. To enable this, we have implemented a query function in CPN Tools that extract a directed graph from the marking of the place “Blocked Objects” describing the waiting conditions between objects. The nodes of the graph represent objects and there is an edge from an object (node)  $o$  to an object (node)  $o'$  if  $o$  is waiting for  $o'$ . We then use Tarjan’s algorithm for computing the strongly connected components of the graph in order to identify cycles. Any strongly connected component containing at least two nodes (objects) represent communication deadlocks (since any two nodes in a strongly connected component are mutually reachable and hence on a cycle). By extracting this directed graph in all reachable markings, we can characterise the communication deadlocks of CPN-ABS (if any). Furthermore, using standard query functions of the model checker we can automatically construct an occurrence sequence starting from the initial marking to any marking containing a communication deadlocks.

If we let  $O$  be an upper bound on the number of objects  $|Obj|$  in the ABS program, then we can extract the directed graph from the marking of “Blocked Objects” in  $\mathcal{O}(O + O^2) = \mathcal{O}(O^2)$  time (in worst case all objects are mutually waiting for each other). As Tarjan’s algorithm is linear in the size of the graph then strongly connected components can be computed in time  $\mathcal{O}(O^2)$ . In the worst case this computation must be done for all reachable markings  $\mathcal{M}_{reach}$  giving a worst case time complexity of  $\mathcal{O}(O^2 \cdot |\mathcal{M}_{reach}|)$  for deadlock detection.

We now use the publisher-subscriber example of Fig. 3 to illustrate how CPN-ABS detects communication deadlocks. By applying the model checker on an Intel i7 3.4 GHz, in less than 1 second we get the full state space report in which tokens of color  $((o_1, Service, q), p)$  and  $((o_3, Proxy, q'), p')$  can be found in the place “Blocked Objects”, and for all  $p, p', q, q'$  we have  $p \notin q'$  and  $p' \notin q$ . This shows that the implementation of the publisher-subscriber protocol is deadlock free.

Now, we slightly modify the protocol, where **get**-statements are added to the method calls in lines 7 and 21 and the **await** statement in line 17 is removed. In this case, CPN-ABS detects a communication deadlock cycle shown in Fig. 13, where  $p \in q'$  and  $p' \in q$  and both objects are trapped in the place “Blocked Objects” and cannot exit from there; in Fig. 13, the third and the fifth argument in the color tuples are outside of the scope of this work, so we ignore them, while,

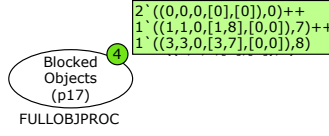


Figure 13: Deadlock detection by CPN-ABS.

```

1 class Client(Service server) {
2   News ns = null;
3   Void run(){server!subscribe(this);}
4   Void pay(){ Fut<Void> f1; f1 = server!subscribe(this); await f1?;
5             Fut<Void> f2; f2 = server!receive(500); f2.get;}
6   Void signal(News n){ns = n;}
7 }
8
9 class Service(Int limit) {
10   Producer prod = new Producer(); Proxy proxy = new Proxy(limit,this,prod);
11   Proxy lastProxy = proxy;
12   Int profit = 0;
13
14   Void run() { this!produce(); }
15   Void subscribe(Client cl){ Fut<Void> f3; f3 = cl!pay(); await f3?;
16                           Fut<Proxy> f4; f4 = lastProxy!add(cl); lastProxy = f4.get;}
17   Void produce(){proxy!start_publish(); }
18   Void receive(Int amount){profit = profit + amount;}
19 }

```

Figure 14: Implementation of the publisher-subscriber example.

the existence of the two zero value tokens is for initialization reasons and they do not affect the deadlock analysis. Based on the information we obtained from this reachable marking, we can trace back to the program code and determine the deadlock represented by the call chain.

Remark that the translation supports scalability: the size of the net is independent from the program and represents the ABS semantics as such. I.e., by increasing the number of `Proxy` objects or clients, only the number of tokens is affected and the analysis is highly automated.

## 7.2. Livelocks

A *communication livelock* is a status of two unblocked objects, in which the generated processes can not progress due to that each of them is busy-waiting for a process on another object. No progress is made at the suspension points, the scheduler is continuously activating and releasing processes. Such a situation may happen, for example, when many processes are competing for entering a critical section. In ABS, process suspension can be done explicitly through the corresponding statement provided by the syntax (**suspend**) or implicitly, through the **await** statement (see Fig. 1 and 2 of Section 2). Activation is an internal procedure, as described in Section 2 with the **ACTIVATE** rule.

The above transitions (suspension and activation) are considered as internal, hence they can be seen as  $\tau$  transitions, since their effect is at the level of the scheduler rather than at the program state. Notice here that, whenever two  $\tau$  transitions (concerning the same object) happen, they should be different, i.e. not both of them SUSPEND or both of them ACTIVATE. This is implied by the semantics of the language (see Fig. 2). For the sake of simplicity, we will not distinguish between them, while alternation between them from now on will be considered as obvious.

A *livelock path for an object* in ABS is an infinite path, where the only transitions that are related to this object are  $\tau$  transitions and there exist infinitely many of them. In ABS, there is a *livelock* if there exist a reachable configuration, such that all infinite paths starting from it are livelock paths for some object and there exists at least such an infinite path.

The CPN-ABS analog of the livelock can be defined in a similar way. Observe that the abstract version  $\tau_\alpha$  of the  $\tau$  transitions described above are the CPN-ABS transitions “Suspend” and “Activate” (labelled as  $a_{17}$  and  $a_{18}$  respectively). They are linked to the places “Busy Objects” ( $p_{15}$ ) and “Idle Objects” ( $p_{16}$ ) and, similarly to the ABS transitions, they alternate since they “move” the object-token from  $p_{15}$  to  $p_{16}$  and vice versa.

In Section 6 we defined the occurrence words over the labels of the transitions of CPN-ABS. So far, whenever obvious, we used to omit the binding from a transition firing. Here, bindings are important, hence occurrence words will be sequences of binding elements, i.e. sequences of pairs consisting of a transition label and the binding  $(a, b)$ . When we are interested in the binding of a subset of the variables of a transition  $t$  we will note it as  $b(s)$ , where  $Var(s) \subseteq Var(t)$ . Below, we provide a definition for *livelocks in CPN-ABS*.

**Definition 14** (Livelocks in CPN-ABS). *A livelock occurrence word for an object  $ob$  is an infinite occurrence word  $\omega$  such that  $\omega|_{ob} = (\tau, b(ob))^\infty$ , where by  $\omega|_{ob}$  we denote the projection of the occurrence word  $\omega$  to the object  $ob$ . Let  $\Omega(M) \stackrel{def}{=} \{\omega \mid \omega \text{ starts from marking } M\}$ . Then, we say that there is a livelock in CPN-ABS iff:*

- $\Omega(M) \neq \emptyset$
- all  $\omega \in \Omega(M)$  are livelock occurrence words for some object  $ob$
- $M \in \mathcal{M}_{reach}$

### 7.2.1. Livelock Detection

As in the case of deadlocks, CPN-ABS is able to detect possible ABS program livelocks like, for example, in the version of the publisher-subscriber example of Figure 14, where the yellow lines induce a livelock: Each client agrees to pay the subscription fee once the server grants the subscription. The server grants the subscription of a client only when the client pays the fee. Both are waiting for each other to act first.

Livelocks in CPN-ABS can be detected based on the state space of the model by exploiting the support in CPN Tools for computing the Strongly Connected Component (SCC) graph of the state space. The SCC-graph has a node for each strongly connected component of the state space containing the states and their connecting arcs, and it has an arc from one SCC  $c_1$  to an SCC  $c_2$  whenever there is a state in  $c_1$  with an outgoing arc to a state in the  $c_2$ . The SCC-graph is a directed acyclic graph and the basic idea in checking for livelocks is to identify states which has a livelock for an object  $o$  by conducting a bottom-up classification of the strongly connected components starting from the leafs (terminal) nodes of the SCC-graph.

A terminal SCC can be classified as a *non-livelock SCC* for an object  $o$  iff it contains arcs corresponding to occurrences of binding elements for other than the suspend/activate transitions for the object  $o$ . A terminal SCC can be classified as a *livelock SCC* for an object  $o$  iff the only occurrences of binding elements related to  $o$  in the SCC correspond to suspend/activate transitions. By construction of the CPN model occurrence sequences (paths) inside such a component will have alternating occurrences of suspend and activate. Hence, all states inside such a strongly connected component will be states that have a livelock for the object  $o$ . Finally, we can classify an SCC as *livelock-neutral* if it does not contain any occurrences of binding elements for the given object.

We can now compute the classification of an SCC based on its outgoing arcs and the classification of its successor SCCs. An SCC is classified as a livelock SCC if:

- The SCC itself only contains occurrence of suspend and activate transitions for the object, all its successor SCCs are either neutral or livelock SCCs, and the arcs connecting the SCC to its successor SCCs concern other objects or suspend/activate for the given object; or
- The SCC does not contain any occurrences of binding elements for the given object, at least one successor SCC is a livelock SCC and the rest are either livelock or neutral SCCs, and the arcs connecting the SCC to its successor SCCs concern other objects or suspend/activate for the given object.

Similar conditions can be obtained for the cases of non-livelock and neutral SCCs and they are similar to how the terminal SCCs are classified.

When the SCC-graph has been computed in CPN Tools the classification can be implemented by exploiting the API in CPN Tools for traversing and querying the SCC-graph. The SCC-graph can be computed in linear time in the size of the state space, and to compute the classification we need to visit each strongly connected component (which is bounded by the number of nodes in the state space) and the arcs inside and between the components (which are bounded by the number of arcs in the state space). If we let  $O$  be an upper bound on the number of objects in the ABS program and  $A$  the number of arcs in the state space, then this gives a worst case time complexity of  $\mathcal{O}(O \cdot (|\mathcal{M}_{reach}| + A))$  for livelock detection.

## 8. Conclusion and Related Work

We have developed an encoding of the formal semantics of ABS as a colored Petri net, such that a program is given as a marking for this net. The key idea in our encoding is to exploit the colored tokens such that our net can support dynamic program behavior and different programs can be represented without making changes to the net structure, but only requires changing the initial marking. We provided a detailed soundness proof for our encoding and showed how a model checker for colored Petri nets can be used for communication analysis of active objects in ABS considering detection of deadlocks and livelocks.

Livelock and deadlock detection is traditionally concerned with the usage of locks for thread-based concurrency. This line of work in the case of deadlocks is surveyed in [24], which develops a type and effect system to capture lock manipulation for such a language. However, in active objects communication deadlocks are caused by call-cycles with synchronization, and the cooperative scheduling of ABS makes the analysis more complex. The problem has been studied using different approaches, including behavioral types [25], cost analysis [26], protocol specifications [23], and Petri nets [17]. Previous work on deadlock analysis for active objects using Petri nets [17] follows

a similar approach such that places represent locks on objects, futures, and processes. Transitions are introduced for each possible caller and callee to a method. To obtain a finite net, the approach abstracts from the actual number of futures such that the wrong future may be accessed in the Petri net. This makes the approach approximative, in that if the net is deadlock free, so is the original active object program. In contrast to these approaches encoding a specific program as a net, our approach directly encodes the language semantics as a CPN and uses markings to define the concrete program; the colors of CPN are used to distinguish different method invocations and to create new objects and the size of the net itself is independent of the specific program. This makes our approach less error-prone and easier to automate as we only need a compiler that given an ABS program generates the corresponding initial marking of our CPN model. Our modelling approach is in this respect similar to the work in [27] and [28], where a CPN model was developed for execution of workflows and for simulation in the planning domain.

Petri nets and its extensions are popular formalisms to model and analyze systems with concurrency, communication and synchronization [14, 15]. Petri nets have in particular been applied to protocol and workflow analysis, but have also been used to study process algebra (e.g., [29, 30]), and more recently including asynchronous communication [31]. CPNs and state spaces have also been used for deadlock analysis of Ada programs in [32], but in contrast to our approach this work did not involve livelock analysis and also employed a non-parametric CPN model. Approaches which encode programming language features into Petri nets have been developed for Ada [16] and more recently for, e.g., Java [33], which focusses on how threads interact with a single synchronized object, and for choreography languages like Orc [34]. CPNs were used in [35] in order to visualise the execution of actor-based concurrent programs. In general, these approaches translate programs into nets such that the size of the program determines the size of the net and dynamic invocations or object creation cause difficulties. Petri nets was also used as a semantic foundation to support a concurrent programming model in [36], and colored Petri nets was used in order [37] to formally define actor semantics.

The work presented in this paper provides several direction for future work. In this paper, we have focused on communication and synchronization for ABS programs. ABS also supports the specification of real-time behavior, deployment architectures, and resource-aware systems [12]. One direction for future work is to extend the CPN model to cover also these language features, and explore the usage of colored Petri nets for resource analysis and to compare resource-management strategies for distributed ABS programs. For the real-time aspects, we may rely on earlier work on scheduling analysis for actor-based system [38]. For the communication analysis we used explicit state space exploration and model checking in its most basic form. For large ABS programs, we will inevitably encounter the state explosion problem during communication analysis. A direction for future work is therefore to investigate state space reduction methods and identify those that are most suited for the domain of ABS program and the communication properties we want to verify. Exploiting symmetries between objects [39] and local progress in the execution of ABS programs [40] are potential candidates in this direction. A third future direction is to further automate our approach by exploring the automatic generation of the initial marking for our CPN model directly from ABS program under analysis, and to be able to visualise any error-traces obtained from the communication analysis at the level of the ABS program being analysed.

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## Appendix A. CPN-ABS Module Details

Here, the interested reader can find more details related to the implementation of CPN-ABS. In addition to the parts of the model presented in Section 4, we provide some extra modules illustrating how the object creation and the communication mechanism of ABS has been simulated with CPNs.

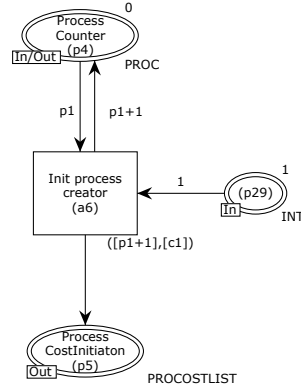


Figure A.15: CPN-ABS module for process creation

In Fig. A.15 we can see the mechanism of the creation of the initial process that is dedicated to each new object upon creation. Place  $p_4$  is a counter with increasing marking, hence we obtain fresh identifiers for the new processes. It is a fusion place, common to the modules related to the communication. Every time a new process is created, its marking is updated, so when firing  $a_6$  we have always a new value  $p + 1$  passed as a parameter to the marking of  $p_5$ . As we explained in the main body of the paper, there are some "extra" variables (here  $c_1$ ) playing the role of a basis for further development of the model. This variable will be related to future cost analysis of ABS programs but for the scope of our current work it is set to zero, hence it can be ignored.

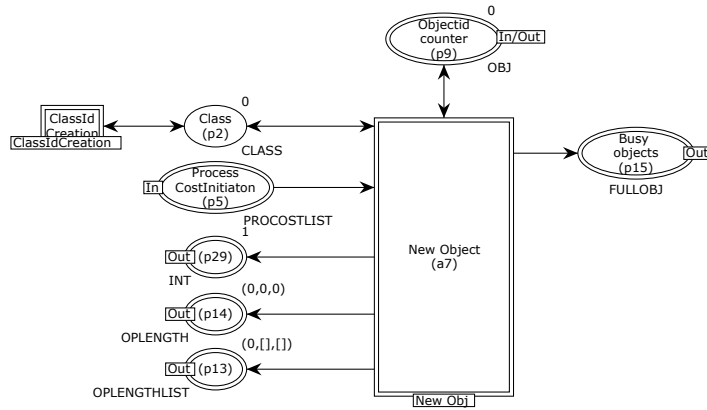


Figure A.16: CPN-ABS module for dynamic object creation (ii)

Figure A.16 shows an upper module of the one related to the object creation (see Fig. 7) which was discussed in Section 4.2. Figure A.17 shows the place which is related to the class identifier. In CPN-ABS classes also are represented as Integers. Transition  $a_1$  creates a fresh identifier for every class ( $M(p_1)$ ) and  $a_2$  and  $a_3$  can change the marking of place  $p_2$ . Remark that the marking of  $p_2$  cannot exceed the value of the last class created. This is useful for binding the class variables of the objects. Similarly for the modules of Fig. A.18 and A.19 referring to the processes and the objects respectively.

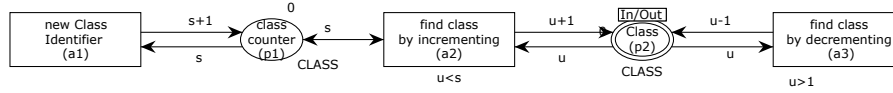


Figure A.17: CPN-ABS module for class identifier creation

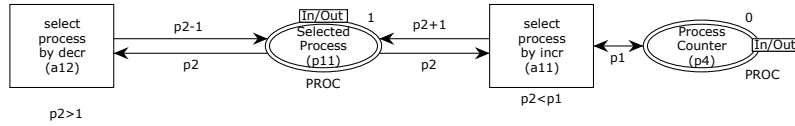


Figure A.18: CPN-ABS module for process selection

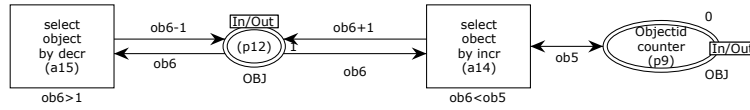


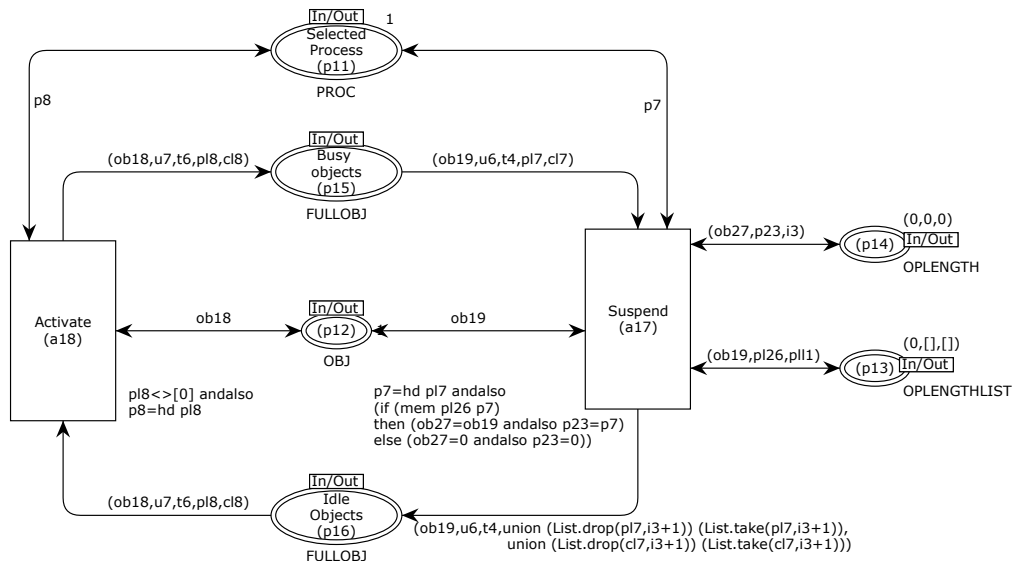
Figure A.19: CPN-ABS module for object selection

In Figure A.20 we can see how the tokens are “moved” from the “*Busy Objects*” place to the “*Idle Objects*” place and vice versa when we have a process suspension or activation respectively.

Figure A.21 illustrates the self synchronous call as it is simulated in CPN-ABS. Every time that  $a_{16}$  fires, a new process is added at the head of the process list of the object (located in “*Busy Objects*” place). In addition, the marking of place  $p_4$  is updated hence the counter is aware of the new process.

In Fig. A.22 we can see in detail how CPN-ABS simulates the return of a method call. As we explained in Section 4.3, when a process related to a method has been executed by an object, then this object becomes idle. Similarly, in CPN-ABS the object token from the “*Busy Objects*” place ( $p_{15}$ ) is removed and it is added to the place “*Idle Objects*” ( $p_{16}$ ) and, on the same time the head of the process list, which represents the active process is removed. In the case that the return of the method resolves a future that is related to a **get** statement, the corresponding process token is produced at place  $p_{23}$ . This is important for binding variables at the tokens of the place “*Blocked Objects*” as we shall see below.

Recall from Section 4 that when an object makes a blocking call (i.e. a call followed by a **get** statement), then the object is moved from the “*Busy Objects*” place to the “*Blocked Objects*” one (Fig. 10). Figure A.23 illustrates how the model matches the resolved future of such a method call and how the object, after that, is moved back to the “*Busy Objects*” place to continue execution.



The color of the tokens of place “*Blocked Objects*” contains not only the object that is waiting for a future, but also the identifier of the corresponding process that has been created for that purpose at the callee object. As we explained above for Fig. A.22, in place  $p_{23}$  there are located only the futures from the methods related to **get** statements. As a result, when an identifier contained in a token of place “*Blocked Objects*” matches with some token of place  $p_{23}$ , then transition  $a_{25}$  can fire and remove the corresponding object from the “*Blocked Objects*” place and add it to the “*Busy Objects*” one. As a result, the (previously) blocked object can continue execution (recall that only the objects located in the “*Busy Objects*” place can execute, otherwise they are either idle or blocked).

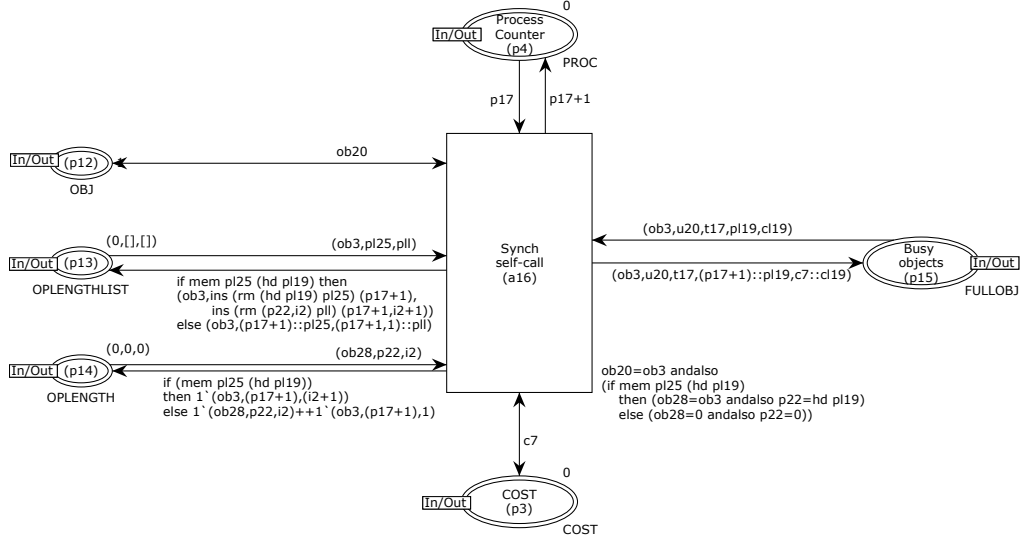


Figure A.21: Module for synchronous self-call in CPN-ABS

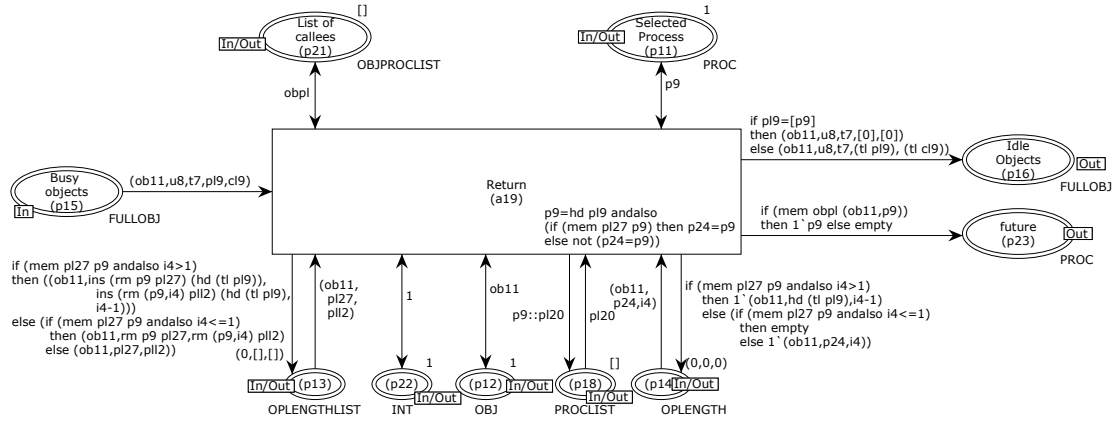


Figure A.22: Module for the return of a method

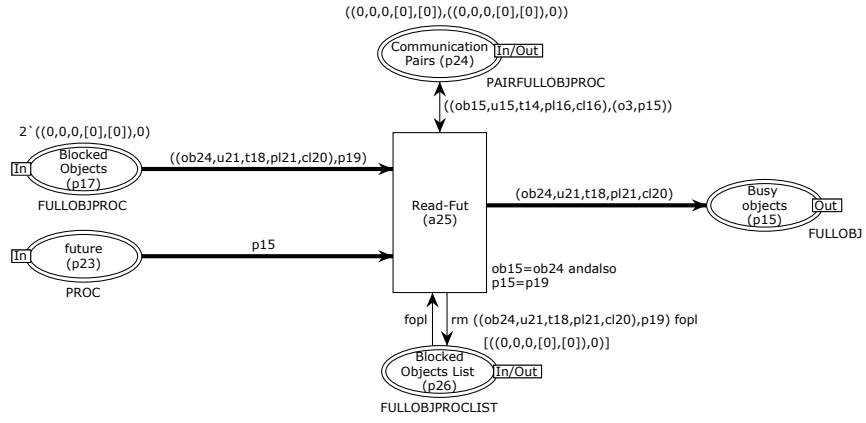


Figure A.23: Module for the future resolution