The Davis-Putnam Procedure

In 1960 the Davis-Putnam procedure was introduced.

The procedure takes as input a formula in Conjunctive normal form, i.e., a set of disjunctions of literals, and tries to show that this is formula is

NOT SATISFIABLE

by applying a set of transformations.

Transformation Rules:

- Tautology simplifications
- Subsumption simplification
- One-literal rule
- Unitary resolution
- Atom elimination

Tautology simplifications

Remove all literals which contain a propositional letter and ist negation, or true or ¬false.

E={p¬pq, rs, ¬rtr, ¬r}

Tautology simplifications

Remove all literals which contain a propositional letter and ist negation, or true or ¬false.

E={p¬pq, rs, ¬rtr, ¬r}

Is transformed into

E={rs, ¬r}

Subsumption Simplification

Remove all clauses which contain other clauses.

$E=\{pq, pqr\neg s, \neg pq, q\neg pr\}$

Is transformed into

$E=\{pq, \neg pq\}$

Subsumption Simplification

Remove all clauses which contain other clauses.

E={pq, pqr¬s, ¬pq, q¬pr}

Is transformed into

E={pq, ¬pq}

Subsumption Simplification

Remove all clauses which contain other clauses.

E={pq, pqr¬s, ¬pq, q¬pr}

Is transformed into

E={pq, ¬pq}

One-literal Rule

Let l be a literal that occurs in a clause in E and such that
$\neg l$ does not occur in any clause.

l is called ***monoform*** lieteral.

Remove all clauses that contain a ***monoform*** *literal*.

E={pqr,¬q¬r, qs, ¬stu, ¬u}

Contains the following monoform literals:

p and t.

Hence, we obtain:

E={¬q¬r, qs, ¬u}

Now, ¬r, s and ¬u are monoform. Finally, we obtain:

E={} which is satisfiable.

Unitary Resolution

A unitary clause is a clause that contains a single literal.

Remove every unitary clause l

and

remove $\neg l$ from each remaining clause.

E={p, q, ¬p¬q}

Remove

p and q

And remove

¬p and ¬q

From each remaining close:

E={nil}

Hence, E is not satisfiable.

Atom Elimination

Let E be a set of clauses without tautologies.

Let P be a propositional letter.

Let T be the clauses in E that do not contain P or ¬P.

Let E´ be the clauses in E that contain P and E´ those which contain ¬P.

Remove from every clause in E´ (resp. E´´)

P (resp. ¬P) leading to NE´ (resp. NE´´).

Combine the clauses in NE´ and NE´´ leading to NE.

Set E to T∪NE.

Eliminate a:

T={bc,¬b¬c},

E´={ac,ab}, E´´={¬a¬c,¬a¬b}.

NE´={c,b}, NE´´={¬c,¬b}.

We obtain: E={bc,¬b¬c,c¬c,c¬b,b¬c,b¬b}.

Tautology eliminates yields:

E={bc,¬b¬c,c¬b,b¬c}.

Elimination of b: E={c¬c,c,¬c}.

Tautology eliminates yields: E={c,¬c}.

Soundness:

Every transformation preserves satisfiability.

Easily checked.

Termination:

take the lexigraphic ordering of the number of propositional letters which occur in E and the cardinality of E as raking function.

It is not difficult to check that it decreases after each transformation.